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# Analysis of Productive Thinking Characteristics Students in The Proofing of Limit Functions

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**Abstract.** The purpose of this research is to describe the productive thinking of prospective mathematics teacher students in solving limit function questions with the concept of epsilon-delta. This research is a kind of descriptive qualitative research carried out in the research program of mathematics education at Universitas PGRI Semarang. The stages carried out consisted of two phases, namely: giving self-regulated learning questionnaires and giving critical and creative thinking test questions. The results of the research with credibility are determined by method triangulation, namely by comparing the results of written tests with interview tests indicating that students with self-regulated learning are thinking critically, and are quite creative having habits of mind with characteristics: can write facts given, namely facts in the form of symbols of the limit function definition, can identify problems in tests comprehensively and can write settlement strategies, can find facts, data, concepts, and can connect them in designing problem solving, especially looking for delta values, can manipulate algebra correctly in searching for delta values even though the answers or delta values obtained have not been written in detail, are not systematic in performing algebraic calculations and manipulations to prove that the selected delta results in a limit function value minus my limit value range from epsilon. In general, students with self-regulated learning are having productive thinking skills even though they still need to be improved, especially in the context of critical thinking.

## INTRODUCTION

Creative thinking produces many possibilities for expanding choices and tends to quickly switch from ideas to ideas, while critical thinking assesses ideas and efforts to focus on things that might give value. Both of these ways of thinking are like two sides of a coin. Although the two cannot be interchangeable, both are much related, bringing a complementary dimension to thinking and learning. Therefore, a new theory of thinking ability appears called productive thinking. The ability to think productively seeks to combine and balance two creative and critical ways of thinking [1,2]. Students must be creative, innovative, enterprising, and adaptable to motivation, confidence, and skills to use critical and creative thinking deliberately [3–5]. Therefore, productive thinking ability which is a combination of critical and creative thinking is needed to be able to survive the competition in life.

The ability to think well will not develop by itself but must be taught [6–8]. The development of thinking power is one of the main goals of education. There are various strategies to explore, develop, and shape one's productive thinking [9]. The ability to think productively is trained and developed through habituation during the learning process. The most important part of learning is helping a person develop thinking skills. This ability will help him learn about what is needed or what he wants to know. This process is termed productive habits of mind. Therefore,

learning is very important emphasizing improving the ability to think productively. Learning is not enough on mastering content. Learning that only emphasizes content will be very easy to forget [1,10].

The main principle of productive thinking is to separate two creative and critical thoughts at the same time. The first step is to think creatively so that you can produce as many choices as possible, then proceed with critical thinking to evaluate and choose the best choice to pursue [1]. Research on increasing students' capacity to think critically and creatively has been done [11]. Mathematics to date (at least taught in schools and colleges) is built on an axiomatic system. In the system, deduction and proof are needed. Real Analysis is one of the courses that can represent this. At the tertiary level, especially for mathematics or mathematics education programs, the Real Analysis course is one of the compulsory subjects. This course is considered by most students difficult because this course requires a high level of logic analysis. Real analysis trains students to think structurally and rationally, which is reflected in the problems posed mostly containing proof. Students must be trained to prove because mathematical competencies, both those proposed by the government, NCTM, and others require reasoning and proof. To eliminate the negative views of students about Real Analysis, the lecturer needs to know the students' ability to think productively students on this course. It aims to be lecturers can apply the order of students ' most thinking abilities, develop learning from students' ultimate understanding when facing problems, information, and new ideas. Moreover, the development of the knowledge of the thinking and practice of using thinking strategies to enhance learning motivation and learning management of students, eventually become problem solvers and thinkers who are confident and independent.

Seeing these conditions, it is necessary to obtain preliminary data regarding the description of the characteristics of students' productive thinking abilities in limit function proof with an approach to epsilon-delta in the eye of real analysis, so that it can be developed as a basis for further research related to the development of learning to enhance productive thinking. The purpose of this research is to describe the characteristics of students' productive thinking abilities in the proofing of limits function with the epsilon-delta approach.

## METHOD

This research is qualitative research, with a primary focus on the current activities of all students solve mathematical problems. The research subjects were 5 prospective mathematics teacher students who had taken Real Analysis courses with moderate self-regulated learning classifications. Data in this research were collected directly by researchers, assisted by supporting instruments. Data collection techniques in this research include written tests and interviews. The first step is to provide the first auxiliary instrument in the form of a Self-Regulated Learning scale consisting of 31 statements in a class and ask all students to do it. The second stage is to provide a second auxiliary instrument which consists of one test of critical and creative thinking. The third step is to check the results of filling the student scale and categorizing Self-Regulated Learning. After that, the researcher checks the results of the critical thinking test and categorizes the level of critical thinking of students following the critical thinking skills gap [12]. In the next stage, the researchers continued to check the results of the work on creative thinking test questions and categorize the level of students' creative thinking following the gap of creative thinking skills [5]. After being categorized, the researcher determined three students from the medium category as the subject of the research to be interviewed at different times.

**TABLE 1.** Summary of productive thinking ability on moderate self-regulated learning

| Initial | Self-Regulation | Express Opinions | | Ability Category |
| --- | --- | --- | --- | --- |
| | | **Writing** | **Verbal** | |
| RM | 66,36 | Unclear | Unclear | Moderate |
| FR | 64,52 | Clear | Clear | Moderate* |
| AAA | 68,66 | Clear | Clear | Moderate* |
| URR | 54,38 | Unclear | Unclear | Moderate |
| FAY | 64,98 | Clear | Clear | Moderate* |

*selected subject

The subjects of this research were prospective mathematics teacher students consisting of five students who had been selected based on the level of self-regulated learning. The selection of research subjects was conducted by consulting with the lecturers in real analysis subjects, to get students who had received the limit function material and were able to communicate answers both verbally and in writing. Based on the results of the consultation, it was found that students who meet the criteria as research subjects as presented in Table 1.

The data analysis technique used is a flow model consisting of three activities that occur simultaneously, namely: data reduction, data presentation, and conclusion [13,14]. In this research to test the validity of the data, a credibility test was carried out because it was the main test in qualitative research. The credibility test in this study was carried out by triangulation, besides that it was also carried out with the Kappa test with the help of the QSR NVivo 11 software. The correlation coefficient categorization is as follows: if the correlation coefficient is less than 0.4 it is said to be poor agreement, the correlation coefficient between 0.4-0.75 is said to be fair to good agreement, and the correlation coefficient is more than 0.75 said to be an excellent agreement [15–18].

## RESULTS AND DISCUSSION

Evaluation of the effectiveness of the actions themselves is usually carried out by students who like to plan and correlate with self-awareness. In understanding the resources needed, one must be able to plan, evaluate the effectiveness of the actions themselves, and have self-awareness. While sensitive to feedback complements existing self-regulation indicators. The strongest relationship on self-regulated learning is planning ability by evaluating the effectiveness with a Pearson correlation coefficient of 0.902239 while the weakest relationship shown in feedback indicators is sensitive with an understanding of the sources needed with a correlation coefficient of 0.388074. Furthermore, the indicators have self-awareness by evaluating the effectiveness of their actions with a correlation coefficient of 0.837367, like planning with self-awareness has the strength of the relationship of 0.814103. Understanding of the resources needed has a relationship with self-awareness 0.715839 while understanding the sources needed by planning only has a correlation coefficient of 0.685723. Sensitive to feedback by evaluating effectiveness and understanding of sources each has a correlation coefficient of 0.44855 and 0.444647. When looking at the similarities between subject indicators that have planning, they can evaluate the effectiveness of their actions, if planning and evaluation of their actions have been carried out then self-awareness will emerge. On the other hand, the understanding of needs with the resources needed is related to other indicators on self-regulated learning except for sensitivity to feedback. This description is also reinforced by Fig. 1 about matrix coding queries in (self-regulated learning).
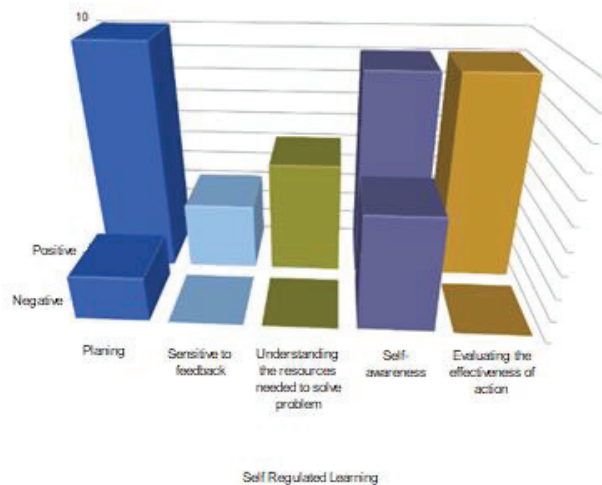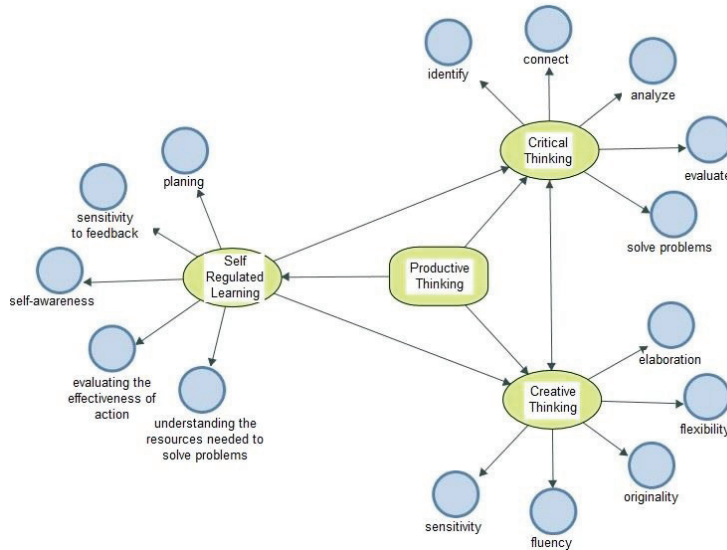


**FIGURE 1.** Project map productive thinking

Furthermore, each indicator of productive thinking includes self-regulation, critical thinking, and creative thinking. Depend on the results of the analysis that productive thinking has three basic bases which include: self-regulated learning, creative thinking, and critical thinking. Critical thinking skills include the ability to identify, connect, analyze, solve problems and evaluate. The ability to think creatively includes the flexibility of thinking, fluency, the authenticity of ideas, having sensitivity, and elaboration in thinking. While self-regulated learning includes the pleasure of a person in planning, being sensitive to feedback, understanding the resources needed to solve problems, evaluating the effectiveness of his actions, and having self-awareness. The results of the research data analysis show that students' productive thinking skills in solving limit functions using the concept of epsilon-delta are seen in Fig. 2.

**FIGURE 2.** Project map productive thinking

Figure 3 shows clearly that three aspects are seen, namely: critical thinking, creative thinking, and self-regulated learning. Among the three aspects, the aspect of critical thinking and creative thinking has the same and greater portion than the aspect of self-regulated learning. This indicates that the role of critical thinking skills and creative thinking is more dominant in solving a problem than self-regulated learning.



**FIGURE 3.** Hierarchical diagram of productive thinking

Figure 4 shows that in distinguish a problem, FAY appears to have a higher percentage than FR and AAA, which is 48.58%, which means that the ability to identify FAY is very good. This is inversely proportional to the ability to connect, evaluate, analyze, and solve problems. The FAY subjects had a low percentage of connecting around 26.67%, in the ability to evaluate around 18.18%, in the ability to analyze around 26.67%, and in problem-solving abilities around 30%. The AAA subjects only can identify 25.71% but can connect by 40%, the ability to evaluate is 45.46%, the ability to analyze is 60%, and the problem-solving ability is 45%. FR subjects can identify as much as 25.71%, the ability to connect is 33.33%, the ability to evaluate is 36.36%, the ability to analyze is 13.33%, and the ability to solve problems is 25%. There is a close relationship between the ability to identify problems by connecting concepts. Both indicators have a correlation coefficient of 0.790469. The ability to identify and connect these concepts greatly supports problem analysis skills. The ability to analyze a problem becomes an important part of solving problems. In solving problems, an evaluation of the answers obtained must be done. This causes there to be a close relationship between problem-solving abilities and the ability to evaluate. Both indicators have a correlation coefficient of 0.666306.
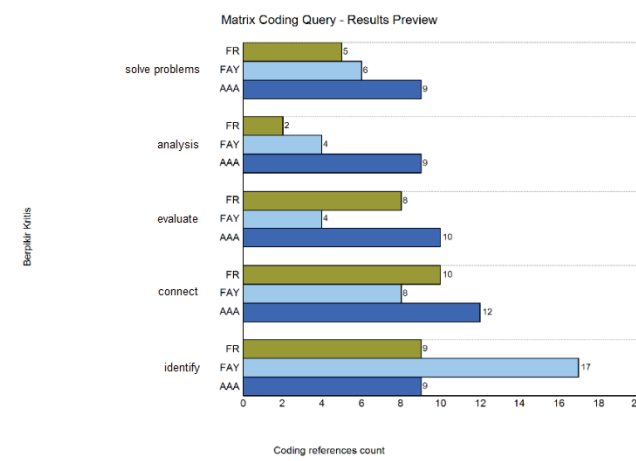
**FIGURE 4.** Indicators of critical thinking

Figure 5 shows that in the sensitivity and smoothness of the FAY, it appears that there are more percentages than FR and AAA, respectively, which are 37.5% and 43.33%, which means that FAY's sensitivity and fluency are very good. This is inversely proportional to the ability of flexibility, authenticity, and elaboration. FAY subjects have a low percentage of flexibility about 20%, authenticity is around 33.33%, and elaboration skills are around 17.65%. The AAA subjects only had a sensitivity of 28.12%, flexibility ability of 36%, fluency of 40%, authenticity ability of 50%, and elaboration ability of 58.82%. FR subjects have a sensitivity of 34.38%, flexibility ability of 44%, fluency of 16.67%, the ability of authenticity of 16.67%, and elaboration ability of 23.53%. There is a close relationship between fluency and flexibility. Both indicators have a correlation coefficient of 0.607496. In solving problems, fluency and flexibility will be seen as integral parts. In addition, there is also a close relationship between elaboration and authenticity. Both indicators have a correlation coefficient of 0.445416. In solving problems, the ability to elaborate greatly influences the originality of the answer.
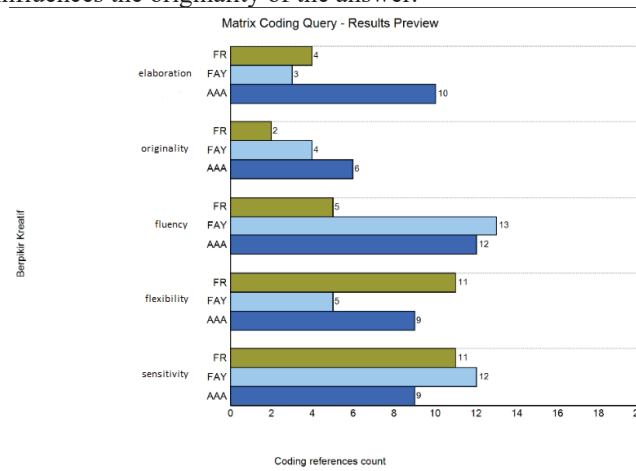


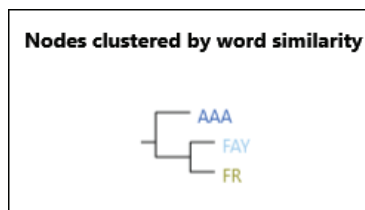**FIGURE 5.** Indicators of creative thinking



**FIGURE 6.** Subjects cluster analysis

**TABLE 2.** Summary of correlations from subject cluster analysis

| Correlation | Coefficient | Category |
|---|---|---|
| FAY-AAA | 0.734137 | Fair to Good Agreement |
| FR-FAY | 0.867854 | Excellent Agreement |
| FR-AAA | 68,66 | Excellent Agreement |

Based on Figure 6 and Table 2, it can be seen that there is a high correlation of information between subjects. This means that the information from the subjects is consistent, which indicates that the research data has been saturated. Furthermore, the level of reliability in this study was measured through the Cohen's Kappa coefficient using the QSR NVivo 11 software. Cohen's Kappa coefficient is better known as the Kappa coefficient. The Kappa test in NVivo's QSR software is adapted from the Kappa test statistics. This test is used to determine the consistency of coding results among research members or research teams. The Kappa coefficient takes into account the number of deals that can be expected to occur by chance. This is the advantage of the Kappa coefficient when compared to the percentage agreement, so many researchers consider the Kappa coefficient to be more useful than the percentage agreement [15–18]. In this study, the kappa coefficient was 0.7885 with an agreement percentage of 97.44%, so that it can be stated that this research has the reliability that is classified as excellent agreement.

The productive thinking ability of 3 students in subjects with self-regulated learning is being, critically and quite creatively. The subject can identify the problem with the test well. The subject can write the facts given, formulate the subject matter of the problem, and can mention the definitions and theorems needed to solve the problem of the limit function problem. The subject can detect the problem in the test well, be able to find facts from the definition of limit functions with the concept of epsilon-delta, and can connect with the algebraic concept in finding the value of the delta. When troubleshooting, subjects can do calculations correctly even though the answers are written are not detailed. The cause of students' weak ability to solve problems is that students are less accustomed to doing the problem-solving process correctly or students are not familiar with non-routine problem problems [7].

In planning completion, the subject cannot provide relevant ideas so that when performing calculations, the answers produced are correct but incomplete. The subject cannot write how to get the results. At that level, students tend to make mistakes and there is no awareness of thinking to investigate to find something "new" [5]. At this level, students cannot fulfill fluency, novelty, or flexibility in solving or raising problems. The problem that many students experience in solving the limit function problem with the concept of epsilon-delta is when searching for delta values that depend on epsilon. Another obstacle to the critical thinking test is when students identify which concepts will be used to solve problems and look for linkages and relationships between concepts. In general, students' productive thinking skills in solving problems of function limits in real analysis subjects have not been seen optimally, this is seen primarily in the ability of self-regulation on indicators of planning and self-sensitivity, while the ability to think critically is seen from students.

**TABLE 3.** Summary of productive thinking ability on moderate self-regulated learning

| Self-Regulated Learning | Critical Thinking | Creative Thinking | Characteristics |
|---|---|---|---|
| Moderate | Critical | Quite Creative | • The subjects can identify test problems well.<br>• The subject can write the facts given clearly, namely facts in the form of symbols from the definition of limit functions.<br>• The subject can identify the problem in the test comprehensively and be able to write a resolution strategy.<br>• The subject can find facts, data, concepts, and can connect them in designing problem solving, especially looking for delta values.<br>• The subject can do algebraic manipulation correctly in searching for delta values even though the answers or delta values obtained have not been written in detail<br>• Subjects that are not systematic in performing algebraic calculations and manipulations to prove that the delta has been chosen result in a limit function value minus its limit value less than epsilon. |

The results of this research indicators show as clearly as creative thinking often do not appear in some subjects with moderate self-regulated learning, especially in the elaboration (see Table 3). Elaboration is part of the habit of creative thinking. Creativity is the act of conscious and unconscious thinking that processes underlies scientific discovery, artistic originality, and inspiration, which has four standard indicators, namely: fluency, sensitivity, flexibility, and elaboration [2,5]. That is, creativity is the act of conscious and unconscious thinking of the processes underlying scientific discovery, artistic originality, and inspiration, which have four standard indicators, namely: fluency, sensitivity, flexibility, and elaboration [3–5,7]. Creative thinking is the ability to see various possibilities in solving problems indicated by fluency, flexibility, originality, sensitivity, and elaboration [19]. Therefore, elaboration is one of the important indicators in creative thinking which one indicator of productive thinking is. Elaboration is starting to solve problems from the general level of moving to a special level so that it can re-grow old knowledge of students. Elaboration is the ability to develop ideas and add or detail the details of an object, idea, or situation so that it becomes more interesting. Elaboration encourages students to expand their ideas and ideas thinking, as well as helping students clarify and articulate their thoughts [8,20]. That is, elaboration encourages students or students to expand their ideas and organize their thinking and helps students or students clarify and articulate their thinking.

## CONCLUSION

Prospective mathematics teacher who has productive thinking ability are students who have above average self-regulated learning, critical and creative thinking skills. The results of the research with credibility are determined by method triangulation, namely by comparing the results of written tests with interview tests indicating that students with self-regulated learning are thinking critically, and are quite creative having habits of mind with characteristics: can write facts given, namely facts in the form of symbols of the limit function definition, can identify problems in tests comprehensively and can write settlement strategies, can find facts, data, concepts, and can connect them in designing problem solving, especially looking for delta values, can manipulate algebra correctly in searching for delta values even though the answers or delta values obtained have not been written in detail, are not systematic in performing algebraic calculations and manipulations to prove that the selected delta results in a limit function value minus my limit value range from epsilon. In general, students with self-regulated learning are having productive thinking skills even though they still need to be improved, especially in the context of critical thinking.

## ACKNOWLEDGMENTS

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