Artikel 32

by Agnita Siska Pramasdyahsari

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Agnita Siska Pramasdyahsari, Rina Dwi Setyawati and Maya Rini Rubowo

Developing Group Theory Textbook which Connected to the School Mathematics' Contents

Agnita Siska Pramasdyahsari [a)], Rina Dwi Setyawati [b)] and Maya Rini Rubowo [c)]

Mathematics Education Department, Universitas PGRI Semarang, Sidodadi Timur St. No. 24, Dr. Cipto, Semarang, Jawa Tengah 50232, Indonesia

[a)] Corresponding author: agnitasiska@upgris.ac.id
[b)] rinadwisetyawati@upgris.ac.id
[c)] mayarinirubowo@upgris.ac.id

Abstract. As part of abstract algebra, the group theory is considered as a difficult subject for pre-service mathematics teachers (PMTs) since it seems not related to the future teaching. In line with the Klein's double discontinuity that the learning of group theory at the university did not bring the content from school mathematics and in the school level mathematics' content did not connected to the university mathematics. Therefore, the current study intends to develop the group theory textbook which connected to the school mathematics that provide the PMTs with the mathematical connection. The study involves the educational research and development cycle refers to Borg and Gall's model, which is adjusted to the need of this study. The research procedures consist of three main stages: collecting data, planning, and developing the product. There is a significant need for further studies to be done in this area to implement this group theory textbook to obtain how PMTs aware to the mathematical connection from school mathematics and vice versa. Furthermore, it is essential to conduct the study about the effectiveness of this textbook. Particularly, in overcoming the PMTs' difficulty in the learning process of group theory subject.

INTRODUCTION

Group theory is one of the subjects required in the Teaching Training Institution (TTI) in Indonesia [1-3] which prepare the secondary pre-service mathematics teachers (PMTs) with the advanced mathematical knowledge (AMK) [4]. However, as part of abstract algebra course, the group theory subject is considered as a difficult subject for PMTs [1-3, 5]. The PMTs are struggling to connect the abstract algebra topic in secondary mathematics for planning their teaching. Nevertheless, several research studies have concentrated on elaborating the mathematical connections between group theory and ring theory to the school mathematics contents [1-3, 6-10]. Moreover, Murray, Baldinger, Wasserman, Broderick, and White [11] identify that the abstract algebra knowledge helps teachers in secondary schools relate approaches, content, and principles to communicating mathematics with their students about the relevance of mathematics. As one example, abstract algebra helps teachers in secondary schools explain commutative properties without using procedural language. In addition, topics in junior high and high school mathematics that are examples of the use of abstract algebra are binary operations, symmetry, solving an equation, inverse functions, and function composition.

The use of mathematical subject matter knowledge which is acquired in undergraduate studies is recognised as the use of the knowledge of mathematical horizon (KMH) when the knowledge is applied to a secondary or elementary school teaching situation [12]. The notion of KMH itself is similar to the idea of the horizon content knowledge (HCK) that is related to the AMK which is defined as subject matter knowledge acquired at colleges or universities [4]. Knowing AMK does not mean that teachers should teach their students with more advanced knowledge. However, this AMK will be considered to become a potential effect in transforming teachers' understanding of school

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mathematics [10]. Even though this particular knowledge does not explicitly explain and discuss to the students, it might influence the instructional approach for teaching. One of the examples from the group theory knowledge is that the terminology such as the inverse and identity element that strengthen and broaden the PMTs understanding of school mathematics content that they will teach [13-14].

Teaching mathematics requires teachers to know what is to be learned and how to explain it. There are topics in secondary mathematics that the teaching can be informed by university mathematics, for example, inverse functions, exponents, limits, trigonometry, numbers, geometry, and polynomials. However, Zazkis and Leikin's [4] indicate that teachers have difficulty in articulating the examples from university mathematics to implement into school mathematics content.

However, the discontinuity curriculum between the university and school mathematics has an impact on teaching practice. Klein [15-16] conceptualised the notion of 'the double discontinuity' curriculum between university mathematics and school mathematics which impacts on teaching practice. The first discontinuity is that the teaching of group theory from university mathematics did not develop from the school mathematics that students (pre-service teachers) have already known. The second discontinuity is the disconnection for the students (pre-service and/or in-service teachers) when they teach school mathematics, group theory appeared unrelated to their future teaching.

To overcome this discontinuity, particularly the group theory subject should provide the connection to school mathematics content explicitly. One of the approaches that can be implemented is that providing the textbook which connected to the school mathematics. Through the textbook the students could are directly involved in the learning process [17]. The textbook becomes one of the main requirements in the learning process [18-19] and the media which become the key elements to increase effectiveness in the teaching and learning process [20]. Moreover, in the mathematics lessons, textbooks can help students build their understanding of mathematics and represent mathematics [19,21]. Since the existence of textbooks is closely related to the curriculum [18, 20]; therefore, mathematics textbooks should support the achievement of competencies that must be mastered.

The assessment of textbooks covers four aspects, namely material, presentation, language and graphics. These four aspects can support the feasibility of textbooks that can be used as an effective and efficient learning support tool [22-23]. The textbooks material is the important aspect that directly affects students' knowledge. The student's knowledge is related to the factual, conceptual and procedural knowledge [24]. Likewise, the material presented in mathematics textbooks should be one of the suggestions for students to learn, thereby increasing students' cognitive and knowledge. Moreover, the textbook's design also could increase the students' learning motivation and reduce students' anxiety levels in learning [25].

Therefore, the researchers intend to look for the solutions to help students having a better understanding to the concept of group theory by providing the mathematical connections through the textbook. The current study aims to develop the group theory textbook which connected to the school mathematics that provide the PMTs with the mathematical connection.

METHOD

The study involved the educational research and development cycle, which refers to Borg and Gall's model [26] in developing the group theory textbook. This model consists of ten procedures: research and information collecting, planning, developing the preliminary form of product, preliminary field testing, main product revision, main field testing, operational product revision, operational field testing, final product revision, dissemination and implementation. However, the present study adjusts the Borg and Gall's model into four main stages: collecting the data, planning, developing the product, and validating and implementing the product. Nevertheless, the current study just presents the three main procedures: collecting data, planning and developing the group theory textbook.

In the collecting data stage, the method used is that compiling the mathematical connection which is linked from group theory and school mathematics. These data were gathered from the previous research about mathematical connection between group theory which involved the PMTs [2]. Furthermore, the curriculum analysis were conducted to check whether the mathematical connection enable to be embedded on the group theory textbook or not.

In the planning stage, the researcher design the group theory textbook which consist of the existing topics from the previous textbook, inserting the context and the using of group theory in the real life, and providing the mathematical connection which is connected to the school mathematics. The design content of the group theory textbook were created by the following arrangement as follows: (a) the definitions of each topics, (b) the related theorems, (c) the proof, (d) the explanation and the elaboration of the concept, (e) the example of solving the problems

related to the topics, (f) the use of topic in the real life, (g) the mathematical connection concept by concept, (h) the exercise followed by the project for the certain topics.

In the developing product, the group theory textbook was designed by considering the previous textbook and the mathematical connection to reveal the concept by concept connection [27-29]. This process involve the rigorous process of selected the mutual concepts from the PMTs identification of mathematical connection. Besides, it also consider the students' conceptual understanding of group theory that vary depending on the mathematical ability [30]. By knowing the various conditions on how students understanding of group theory, it will assist the researchers to determine the difficulty level which can be applied for all the students. Therefore, the textbook could provide the heterogeneous students to use the textbook easily.

RESULT AND DISCUSSION

In this study, the group theory textbook was developed through three stages of procedures adjusted from Borg and Gall's model [26]. The first stage is that collecting data. The data was collected from compiling the mathematical connection which is linked from group theory and school mathematics. The result of this stage is the compilation of the mathematical connection concept by concept from the previous research finding [2] which is in line to the Suominen and Usiskin [27-29]. The mathematical connection linked from group theory and school mathematics will be presented on the Table 1.

TABLE 1. Mathematical connection concept by concept per chapter.

Group Theory Concept	Secondary School Mathematics Concept
Group and its Properties	Identity, Inverse, Function and domain, Solving linear equations, Number systems and operators
Binary Operation	Arithmetic operations and number systems, Domain, Function, Transformation of function, Composition of function
Subgroup	Subsets
Theorem Lagrange	Least Common Factor, Great Common Factor
Permutation group and multiplication of cycle	Composition of function, permutation, symmetry
Cyclic group	Division of algorithm, Great Common Factor, rotation
Homomorphism/isomorphism	Equality, Function, Mapping
Inverse	Multiplicative reciprocal Negative numbers

Meanwhile, in the planning stage, the data obtained were the design of structure and contents of the group theory textbook which is elaborated as follows: (a) the definitions of each topics, (b) the related theorems, (c) the proof, (d) the explanation and the elaboration of the concept, (e) the example of solving the problems related to the topics, (f) the use of topic in the real life, (g) the mathematical connection concept by concept, (h) the exercise followed by the project for the certain topics.

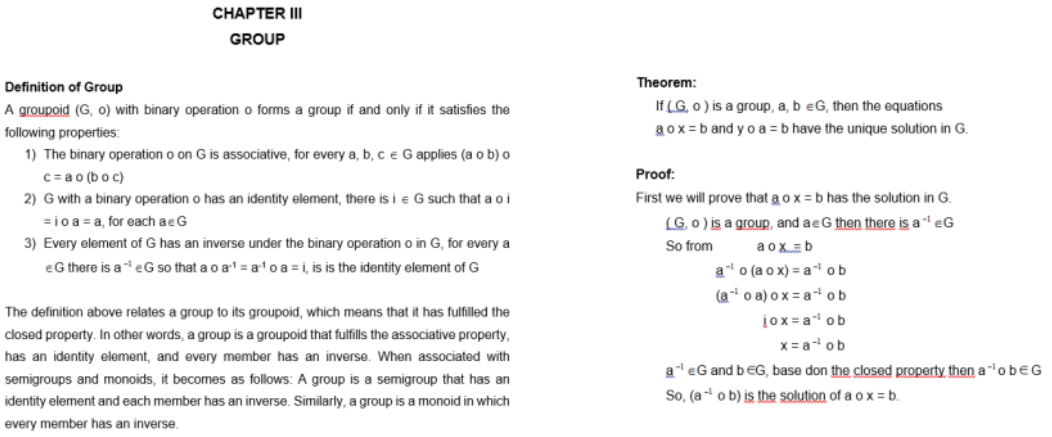
In the textbook structure of 'definition of each topic', it provides the formal definition which is explained explicitly the topic followed by the elaboration and the examples. This definition will be followed by 'the theorems related to the topics and the proofing process.' This structure aimed to provide the PMTs with the rigorous procedures of proofing. However, this design does not limit the PMTs creativity to use the different procedures. After showing the related theorem, the textbook structure will be continued by 'explaining and elaborating the concept' comprehensively by providing the more complex ideas. Then, it followed by 'the example of solving the problems related to the topics'. Through this example PMTs could obtain an overview to grasp the idea to solve the similar problems related to the discussed topics. The next structure is showing 'the using of the topic in the real life'. The purpose of this design is to inform the PMTs how the concept in group theory could support other fields and give the benefit to the development of knowledge.

The new appearance in the recent group theory textbook is providing 'the mathematical connection concept by concept'. This part is not available in the previous textbook, therefore it becomes the special textbook design which accommodate the need of mathematical connection to overcome the Klein's double continuity [15-16]. The existence of this mathematical connection will inform and give the PMTs an overview how each topics in the group theory are connected to the school mathematics content. Moreover, it could motivate and increase the awareness of PMTs to use the knowledge from group theory in explaining the school mathematics concept. Furthermore, it is expected that the

mathematical connection that PMTs know will influence the instructional approach for teaching and transform its ideas into mathematics school content. This idea is in line to the Wasserman [10] that the advanced mathematical knowledge acquired from the university mathematics will equip the PMTs with the subject matter knowledge which can apply and transform into secondary mathematics teaching that considered as knowledge of mathematical horizon [12].

The last structure in the textbook is that completing the topic with the exercise and the project for the certain topics. The difference with the previous textbook is the adding of particular project based learning for any topic. The purpose of giving the exercise is to evaluate how the PMTs grasp the concept and provide them with the opportunity to reflect how they can master the topic. Moreover, it also enable the heterogeneous course participants with various conceptual understanding of the topic and with different level of mathematical ability to have better understanding the concept of group theory. Meanwhile the project will be conducted to allow the PMTs doing the activity which involve their previous knowledge and present knowledge to be collaborated in designing the instructional design for their future teaching in the secondary mathematics setting.

Furthermore, in the developing procedures, the group theory textbook was designed by considering the previous textbook and the mathematical connection to reveal the concept by concept connection. The example of each structures from the new textbook's design will be displayed on the following figure. Figure 1 show the structure definition part (a) and the theorem and its proof. It provides the definition of group, theorem and its proof from chapter III of the new design of group theory textbook that has been translated into English for publication purpose.



CHAPTER III
GROUP

Definition of Group

A groupoid (G, o) with binary operation o forms a group if and only if it satisfies the following properties:

1) The binary operation o on G is associative, for every a, b, c ∈ G applies (a o b) o c = a o (b o c)
2) G with a binary operation o has an identity element, there is i ∈ G such that a o i = i o a = a, for each a∈G
3) Every element of G has an inverse under the binary operation o in G, for every a ∈G there is a^{-1} ∈G so that a o a^{-1} = a^{-1} o a = i, is is the identity element of G

The definition above relates a group to its groupoid, which means that it has fulfilled the closed property. In other words, a group is a groupoid that fulfills the associative property, has an identity element, and every member has an inverse. When associated with semigroups and monoids, it becomes as follows: A group is a semigroup that has an identity element and each member has an inverse. Similarly, a group is a monoid in which every member has an inverse.

Theorem:

If (G, o) is a group, a, b ∈G, then the equations
a o x = b and y o a = b have the unique solution in G.

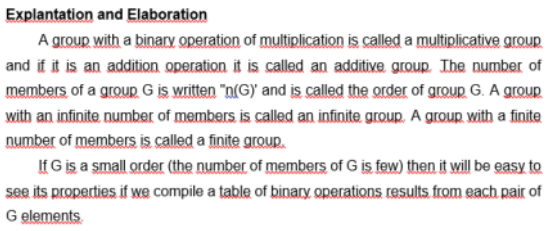
Proof:

First we will prove that a o x = b has the solution in G.
(G, o) is a group, and a∈G then there is a^{-1} ∈G
So from a o x = b
a^{-1} o (a o x) = a^{-1} o b
(a^{-1} o a) o x = a^{-1} o b
i o x = a^{-1} o b
x = a^{-1} o b
a^{-1} ∈G and b∈G, base don the closed property then a^{-1} o b ∈ G
So, (a^{-1} o b) is the solution of a o x = b.

(a) (b)

FIGURE 1. The textbook part of definition, theorem and its proof.

The following part is displayed on the Fig. 2, that is the 'the explanation and the elaboration of the concept' this part is similar to the previous design since it is the main idea that should be delivered.



Explantation and Elaboration

A group with a binary operation of multiplication is called a multiplicative group and if it is an addition operation it is called an additive group. The number of members of a group G is written "n(G)' and is called the order of group G. A group with an infinite number of members is called an infinite group. A group with a finite number of members is called a finite group.

If G is a small order (the number of members of G is few) then it will be easy to see its properties if we compile a table of binary operations results from each pair of G elements.

FIGURE 2. The explanation and elaboration of concept group.

The next Fig. 3 show the example of solving the problems related to the topics, and the exercise part. As the purpose of example and exercise to help PMTs grasp the concept; therefore, it should provide the problem that has

already discussed through the example. So, it could minimize the number of unrelated problem and avoid the evaluation that is not suitable to the topic.

Example:

M = { 0, 1, 2 } is a set of integers modulo 3. All the multiplication of integers modulo 3 was displayed on the table below. Investigate whether M is a group or not.

Multiplication Table

x 3	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

(M, x3) is not a group because 0 does not have invers. It can be seen from the first row is 0.

(a)

Exercise Part

1. Write the definition of grupoid, semigrup, and monoid.
2. Write the definition of group and give 5 examples of group.
3. Complete the table below

   A = $\{-1, 1, -i, i\}$ with i= $\sqrt{-1}$, Investigate and explain whether (A, x) is a group or not.

X	1	-1	i	$-i$
1				
-1				
i				
$-i$				

(b)

FIGURE 3. The example and exercise part.

Figure 4 shows 'the use of topic in real life' part, which provides the application of symmetry group in technology (a) and the mathematical connection concept by concept (b).

The Application of Symmetry Group in the Field of Technology

The batik design software was designed by Muhamad Lukman, Nancy Margried and Yun Hariadi by combining technology, architectural art and mathematics which then produces what is called **fractal batik.** Then they formed a team called the Pixel People Project which is engaged in art and design. With the ***jBatik*** design software, the process of making batik patterns can be done by mapping batik patterns mathematically which will then be translated into digital batik patterns by this batik software. Even just one pattern when combined with other patterns in ***jBatik*** can produce new batik patterns. jBatik is able to carry out various functions to make batik patterns as if making batik patterns with canting, as follows:

(a)

Group Theory Concept	Secondary School Mathematics Concept
	Transformation of function
	Arithmetic operations and number systems
Subgroup	Subsets
Theorem Lagrange	Least Common Factor, Great Common Factor
Permutation group and multiplication of cycle	Composition of function, permutation, symmetry
Cyclic group	Division of algorithm, Great Common Factor, rotation
Homomorphism/isomorphism	Equality
	Function
	Mapping
Inverse	Multiplicative reciprocal
	Negative numbers

(b)

FIGURE 4. The application and the mathematical connection part.

Considering the new design of the group theory textbook, it is expected to fulfil the standard of content material, the presentation, language and the graphics which is easy to be followed by the PMTs that support the feasibility of textbook that can be used as an effective and efficient learning support tool as stated by BSNP [22]. However, it is needs for further study to know how effective the new design affected to the PMTs conceptual understanding and the awareness of the mathematical connection to the school mathematics

CONCLUSION

The group theory textbook which is connected to the school mathematics is aimed to provide the PMTs the concept of group theory with the mathematical connection concept by concept. This textbook was designed by considering the presenting the mathematical connection concept by concept which is also supported by the content arrangement. The group theory textbook's design consisting the definition, theorem, proof, explanation and elaboration, the real life connection, the concept by concept mathematical connection and the exercise following by the project for certain topics. There is a significant need for further studies to be done in this area to implement this group theory textbook to obtain how PMTs aware to the mathematical connection from school mathematics and vice versa. Furthermore, it is essential to conduct the study about the effectiveness of this textbook. Particularly, in overcoming the PMTs' difficulty in the learning process of theory group.

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