Artikel 27

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Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

Developing the scripting task for mathematical connection between the university and school mathematics content

To cite this article: A S Pramasdyahsari 2021 *J. Phys.: Conf. Ser.* **1957** 012003

Developing the scripting task for mathematical connection between the university and school mathematics content

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Abstract. The pre-service mathematics teachers (PMTs) found abstract algebra is difficult and irrelevant to the secondary teaching. Even though several studies have elaborated the connection between them, they cannot explicitly use its connections. Therefore, the study aimed to describe the development of a scripting task design that encourages the secondary mathematics pre-service teacher to use the mathematical connection from abstract algebra to solve the school mathematics problem. The study involves the educational research and development cycle refers to Borg and Gall's model, which is adjusted to the need of this study. The research procedures consist of three main stages: collecting data, planning, and developing the product. In designing the scripting task, several task principles were involved, such as necessity, two-way synergy, interest, focus, awareness, and self-reflection. The scripting task was developed through at least two considerations that are accomplishing both professionally relevant and mathematically rigorous goals and the relevance of topic for both the university and school mathematics. There is a significant need for further studies for implementing this scripting task to see teachers' awareness in using the mathematical connection from university mathematics to help their students solve school mathematics problem.

1. Introduction

The study of mathematical teachers' knowledge has received considerable attention [1-2]. Ball and her colleagues [1] developed a model to identify what kind of knowledge is required for a mathematics teacher to know in order to teach effectively. They developed a particular framework which describes mathematical knowledge for teaching (MKT). Several studies have investigated the significance of MKT in a classroom setting [1, 3-7]. In general, this knowledge prepares teachers to make the subject understandable to the students. Vale et al. [7] argue that having MKT enables teachers to broaden and deepen their knowledge to support students' present and future learning of mathematics and interpreting and developing students' ideas. In line with this argument, even [3] emphasizes that the comprehension of MKT will create a learning environment which promotes the development of students' mathematical power, and it is significantly related to students' achievement.

The use of mathematical knowledge for teaching especially advanced mathematical knowledge (AMK) is still debatable among stakeholders [8-9]. Zazkis and Mamolo [10] argue that teachers' knowledge at the mathematical horizon includes advanced mathematical knowledge and a broader understanding of the connection that contributes to teaching instruction and benefit for students' learning. However, having AMK does not mean teachers can use it directly in their classroom teaching. The AMK provides teachers with a broad understanding of basic mathematics which enables them to give students mathematical experiences beyond the mathematics syllabus [11-12]. Zazkis and Zazkis [13] responded this argumentation by stating even though advanced mathematical knowledge is not the primary aspect for pre-service teachers but the use of this knowledge can help them to develop the value of awareness, to deal with the problem-solving task, and to get outside from confusion. The AMK impacts teachers' work in the classroom both in practice and in the plan [12, 14]. It possibly supports teachers to make any alteration or addition in instructional plans. Moreover, the AMK also helps teachers conceptualise and structure mathematics which facilitates them to connect with mathematics content in secondary teaching [11, 15].

Abstract algebra is one of the courses acquired from the university mathematics curriculum; therefore, it is referred to as one of the advanced mathematical knowledge. Dubinsky, Dautermann, Leron, and Zazkis [16] argue that the abstract algebra course contributes to the teachers' mathematical knowledge in providing an opportunity to understand the structure of number systems. Abstract algebra provides opportunities for pre-service secondary mathematics teachers to develop concept images and concept definitions [17] that help them to explain the concepts in school mathematics. Moreover, undergraduate students also learn the terminology and methodology of algebra through abstract algebra [18].

As a part of AMK, which is required for mathematics preservice teachers, abstract algebra is difficult and irrelevant because of the disconnection between the university and school mathematics curriculum [19]. Wasserman [20] identifies the specific topics which connect abstract algebra to the teaching of arithmetic properties, inverses, the structure of sets, and solving equations. These findings attempt to address the second part of Klein's 'double discontinuity' [21] that abstract algebra can relate to school mathematics teaching. However, it is difficult to find the literature which establishes how teaching connection from abstract algebra would enable the teachers to be aware of using abstract algebra in their teaching practice. The remaining question is how to develop the task that encourages teachers to use the teaching connection in the classroom.

Zazkis, Sinclair, and Liljedahl [22] introduced a scripting task as a lesson play, where research studies in mathematics education have used a scripting task to explore pre-service and in-service teachers' conception in teaching mathematics [23-25]. The scripting task provides both pre-service and in-service teachers with an opportunity to envision students' difficulties both mathematically and pedagogically [24]. Following this argument, Zazkis and Kontorovich [25] suggest that the using scripting task can connect the idea from advanced mathematics to a classroom situation which encourages to enhance teachers' mathematical knowledge. Therefore, the present study attempts to develop a scripting task for teachers that encourages secondary mathematics pre-service teachers to use the teaching connection from abstract algebra as a form of advanced mathematics to solve the school mathematics problem.

2. Methods

The study involved the educational research and development cycle, which refers to Borg and Gall's model [26] in developing the scripting task [27]. This model consists of ten procedures: research and information collecting, planning, developing the preliminary form of product, preliminary field testing, main product revision, main field testing, operational product revision, operational field testing, final product revision, dissemination and implementation. However, the present study adjusts the Borg and Gall's model into four main stages: collecting the data, planning, developing the product, and validating and implementing the product. Nevertheless, the current study just presents the three main procedures: collecting data, planning and developing the scripting task.

In the collecting data stage, the method used is literature review and analysis to determine the task, which enables the researcher to know how the mathematics pre-service teachers will involve their mathematical content knowledge into an imaginary dialogue between student and teacher. In this stage, the researchers decided to use the inverse of trigonometric function problem from school mathematics school. This problem refers to the mathematical connection [28-29] connect to abstract algebra, particularly the concept of inverse both in group and ring.

In the planning stage, the researcher decided the type of task which accommodate the purpose. Since the purpose is to encourage the secondary mathematics pre-service teachers to use the teaching connection from abstract algebra in solving the school mathematics problem; therefore, the scripting task as a lesson play [22] was designed. The consideration of choosing this kind of task is in line with the function of the scripting task, which enables to explore pre-service and in-service teachers' conception in teaching mathematics [23-25].

While designing the prompts, the researcher decided to create a scripting task that considers several considerations related to the experiences within the teaching and learning process as a student and a lecturer. In the developing product, the scripting task was designed using five principles regarding the needs to accomplish the purpose of both professionally relevant and mathematically rigorous, and the relevance of topic for both the university and school mathematics.

3. Result and discussion

This part firstly presents the result of developing the scripting task, and the second part discusses the results.

3.1. Result

In this study, the scripting task was developed through three stages of procedures adjusted from Borg and Gall's model [26]. The first stage is that collecting data. The data was collected from the literature review to analyse the topics connected to university mathematics and school mathematics. The result of this stage is choosing the invers of trigonometric function as the topic involved in the scripting task.

Meanwhile, in the planning stage, the data obtained were the considerations for designing the scripting task and the task principles. The scripting task was designed by following the two requirements: accomplishing the goals of both professionally relevant and mathematically rigorous and the relevance of topic for both the university and school mathematics. This consideration is supported by the previous research that in the teaching preparation program, to overcome Klein's double discontinuity [21] between university and school mathematics, therefore, the lecturers should provide the teaching resource which facilitates the mathematics pre-service teachers to recognize the connections of the university mathematics and school mathematics [28].

Moreover, there are several principles involved in designing the task as follows necessity, two-way synergy, interest, focus, awareness, and self-reflection. The *necessity* principle means the task should be necessary for both students and teacher to "see the need for mathematics we intend to teach them" [29]. The *two-way synergy* principle means the task should accomplish professionally relevant to school mathematics teaching and mathematically rigorous from advanced mathematics knowledge. This principle aligns with the designing task principle 'building up from and stepping down to practice' [30]. The *interest* principle means the task should be adequately interesting to promote engagement. The *focus* principle means the task should enable the participants to choose mathematical ideas and minimise unrelated ideas. Then the awareness principle means the task should provide an opportunity for the participant to trigger their awareness of using the knowledge that they have before, and the last *self-reflection* principle means the task should provoke the participants to do self-reflection to justify their thinking to themselves and elaborate the explanation to help overcome the students' confusion.

Furthermore, in the developing procedures, the task was designed a script as a lesson play which provides the dialogues between teacher and the students in the classroom teaching setting. In this stage, the scripting task was designed to be assigned to the pre-service mathematics teachers (PMTs) based on the task principles. The PMTs asked to continue an imaginary dialogue between a teacher and two students (a group of students) in which the teacher explains to their students to help overcome students' confusion about the inverse of sine functions. This task is called a scripting task which is introduced by Zazkis, Sinclair, & Liljedahl [22] as a lesson play.

For the preliminary field testing, this task was distributed to the PMTs who took algebra class. The PMTs asked to present a script's dialogue for interaction between students and teacher related to trigonometric form. The scripting task which is developed will be presented on the figure 1 as follows.

You are given the beginning of an imaginary dialogue between a teacher and two students (or several students) in a secondary school setting. You are asked to continue the dialogue abov e and provide the solution of the question 'are any of $-\sin(x)$, $\csc(x)$, and $\sin^{-1}(x)$ the same? '.

Mrs. Riana :	*Consider these three-trigonometric functions* $-\sin(x)$, $\csc(x)$, *and* $\sin^{-1}(x)$. *Are any the same?*
Jono :	*Is* $-\sin(x)$ *equal to* $\sin^{-1}(x)$*?*
Mrs. Riana :	*Why do you think that, Jono?*
Jono :	$\sin^{-1}(x)$ *means the inverse of* $\sin(x)$. *The inverse of* $\sin(x)$ *should be* $-\sin(x)$. *Therefore,* $\sin^{-1}(x) = -\sin(x)$
Cindy :	*I don't think so.*
Mrs. Riana :	*What do you think, Cindy?*
Cindy :	*I think* $-\sin(x)$ *is different to* $\sin^{-1}(x)$.
Mrs. Riana :	*Why do you think they are different to each other?*
Cindy :	*I do agree with Jono that* $\sin^{-1}(x)$ *means the inverse of* $\sin(x)$. *But, the inverse of* $\sin(x)$ *which is denoted as* $\sin^{-1}(x)$ *is* $\csc(x)$.
Mrs. Riana :	*I understand both of your arguments, Jono and Cindy.*
Jono :	*So, which one is correct?*
Mrs. Riana :	*Let's consider what the inverse element of the set and the inverse of function are.*
Cindy :	*Well, suppose I have an element from a set of integers such as 5 and the inverse of 5 is -5 is it what do you mean by inverse element?*
Mrs. Riana :	....

Explain your choice of approach, why did you choose a particular example to address your students' confusion? Why do you think that your explanation is appropriate for your students?

Figure 1. The product of scripting task design.

The purpose of the preliminary field testing is to acquire the description from the PMTs related to the student's mistake with the reason and how the PMTs play as a teacher to address the error to help their students solving the problem. Finally, the scripting task could be a task to analyse teachers' knowledge, either mathematical or pedagogical. The figure 2 is one of the examples of the preliminary field-testing result. It shows the extended dialogue from the teacher and several students to continue the provided dialogue.

Mrs. Riana	: Yes, that's right, Cindy, why is that? What do you think Jono?
Jono	: Suppose I have a real number member like 3 and an inverse of 3 which is $\frac{1}{3}$. Is that also an inverse element mam?
Mrs. Riana	: Yes, really Jono. Wow, thank you Jono and Cindy, you are great. Then why is it an inverse element?
Dea	: Ma'am, I want to give an opinion, from Cindy's answer if 5 + -5 = 0 and from Jono's answer $3 \times \frac{1}{3} = 1$. Whereas 0 and 1 are elements of identity. Does the inverse element have anything to do with the identity element?
Mrs. Riana	: Well, thank you Dea for your opinion. Then what is the conclusion of the inverse element and is there a relationship between the inverse element and the identity element? Let's consider Dea's opinion on the nature of the binary operation about the inverse element if $\forall\, a \in S, \exists a^{-1} \in S$ such that $a * a^{-1} = a^{-1} * a = e$ where * is binary operations whereas a is a member of the set of real numbers and e is an element of identity. And for the multiplication operation with members of the set of real numbers, it has an identity of 1, while for the addition operation with members of the set of real numbers, it has 0 as an identity.

Figure 2. The excerpt of scripting task from the preliminary test.

By considering the result of the preliminary field test, the scripting task has already occupied the purpose to acquire the description from the PMTs related to the student's mistake with the reason and how the PMTs play as a teacher to address the error to help their student solving the problem. Therefore, there is no modification needed after the preliminary field test.

3.2. Discussion

As the scripting task was introduced as a lesson play [22], the scripting task was also designed to enable the PMTs to play a role both as a secondary school student and a teacher in the developing product stage. Several research studies in mathematics education have used scripting tasks to explore pre-service and in-service teachers' concept in teaching mathematics [23-25].

In the first stage, the inverse of trigonometric function was chosen based on the obtained literature regarding the mathematical connection between university mathematics and school mathematics. This topic was chosen because of the most common confusion in determining the inverse of a function [31]. As cited from Wasserman [31] that the first confusion relates to the common procedure of switching x and y then solving y. Second, the difficulty and confusion about the students' familiarity with an arithmetic operation such as addition and multiplication rather than the composition of function which is viewed as not an operation. Another confusion that Wasserman [31] identifies is the use of notation $^{-1}$ as homonymy [25], the same name for different things, which is used to represent the reciprocal. Okur [32] claims that students have confusion about the symbol $f^{-1}(x)$ as it represents both the inverse relation and the inverse function. He emphasises that students' confusion about inverse functions should be overcome by explicitly distinguishing between the concept of the inverse function and the inverse relation, which is demonstrated better through a particular abstract algebra concept. Teachers who have a deeper understanding of algebraic structures in abstract algebra would have the opportunity to develop an explicit definition of the inverse in relation to a set, operation, and identity element [33].

Meanwhile in the planning stage, designing the scripting task and the task principles hold an important role. It determines how the task could accomplish the purpose itself to encourage secondary mathematics pre-service teachers to use the teaching connection from abstract algebra as a form of advanced mathematics to solve the school mathematics problem**.** From the scripting task, it shows the *necessity* principle that the topic related to the inverse trigonometric function is necessary for both students and teacher since it is an important topic to be grasped. Moreover, this scripting task which focus on inverse trigonometric function is relevant to the school mathematics teaching and taking the mathematically rigorous from advanced mathematics knowledge of inverse element from group theory. Therefore, it is fulfilled the *two-way synergy* principle. Furthermore, it is shown from the preliminary field test (see figure 2) that this problem is also adequately interesting to promote engagement, which mean that the *interest* principle is fulfilled. The task also enable the participants to choose mathematical ideas and minimise unrelated ideas (*focus* principle) and it provides an opportunity for the participant to trigger their awareness of using the knowledge of inverse element in group theory that they have before (*awareness* principle). At last, the scripting task provokes the participants to do self-reflection to justify their thinking to themselves and elaborate the explanation to help overcome the students' confusion (*self-reflection* principle). Therefore, the task fulfils the requirement both professionally relevant and mathematically rigorous, and relevant topic for both university mathematics and school mathematics.

Regarding the third stage, the development of the scripting task was designed as a lesson play which guides the PMTs to imagine a learner who might experience difficulties or confusion, predict the challenges or the student's answer, and help the student to overcome the challenges by explaining to the student through the dialogues. Zazkis, Sinclair, & Liljedahl [22] argued that asking the PMTS to think about their future teaching in terms of imaginary interactions draws their attention to how the students will develop mathematics thinking. However, the scripting task also provide in-service mathematics teachers (IMTs) to imagine how the interaction between a teacher and their students is explained through the dialogues. This is in line with Pramasdyahsari, Setyawati, and Albab [34] which reveal that the IMTs could recall and create the mathematical connection between university mathematics and school mathematics, in case of group theory and school mathematics by providing them teacher professional development.

Furthermore, the scripting task provides both PMTs and IMTs with an opportunity to envision students' difficulties both mathematically and pedagogically [24]. Following this argument, Zazkis and Kontorovich [25] suggest that scripting tasks can connect the idea from advanced mathematics to a classroom situation which encourages to enhance teachers' mathematical knowledge. In particular, the

Journal of Physics: Conference Series **1957** (2021) 012003 doi:10.1088/1742-6596/1957/1/012003

teachers' (pre-service or in-service) mathematical knowledge that involved becomes apparent in their attempts to overcome students' confusion.

4. Conclusion

The scripting task is aimed to encourage the mathematics teachers to use the teaching connection from the knowledge of abstract algebra in solving secondary mathematics problems. This scripting task was designed by choosing the idea of the trigonometric inverse function. It is developed through at least two considerations that are accomplishing the goals of both professionally relevant and mathematically rigorous, and the relevance of the topic for both the university and school mathematics. The design prompt of the scripting task uses several task principles: the necessity, two-way synergy, interest, focus, awareness, and self-reflection. There is a significant need for further studies to be done in this area to implement this scripting task to see teachers' awareness in using the mathematical connection from university mathematics to help their students solve school mathematics problems. Moreover, it can be one of the suggestions for other colleagues to develop the scripting task for other advanced mathematical knowledge from university mathematics.

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