PROBLEMS OF EDUCATION IN THE 21ST CENTURY

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PROBLEMS OF EDUCATION IN THE 21st CENTURY

TITLE OF ARTICLE:

EXPLORING OF MULTI MATHEMATICAL REPRESENTATION CAPABILITY IN PROBLEM SOLVING: STUDY ON SENIOR HIGH SCHOOL STUDENTS IN INDONESIA

ID: 2_SAL-sa_review PEC_30CFP Reworked (new evaluation in red) 29CFP

P

1.	Is the article original, and does it contribute something new to the field? (Importance of article / Relevance and Appeal to national / international scholarly community)	**Excellent**	Good	Moderate	Poor
2.	Statement of problem (s) / aim (s) / objective (s)	Excellent	Good	**Moderate**	Poor
3.	Theoretical basis / Theoretical framework / Literature review / Clarification of concepts	Excellent	**Good**	~~Moderate~~	Poor
4.	Appropriateness of the research plan and design (if applicable) /Appropriateness of data-collection and procedure /Data analysis /Trustworthiness/ reliability and validity	Excellent	Good	**Moderate**	Poor
5.	Steps taken to ensure that the research complies with standard ethical guidelines (if applicable)	Excellent	Good	**Moderate**	~~Poor~~
6.	Data presentation / Discussion (Are the results clearly and correctly presented? Are they consistent with the methodology?)	Excellent	**Good**	Moderate	Poor
7.	To what extent is the line of argumentation in the article clear, cohesive and logical?	Excellent	**Good**	Moderate	Poor
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10. Please write a brief narrative report on the article in which you provide a general or overall assessment of the manuscript and its suitability for publication.

General comments:

- ~~The paper Cannot be accepted for publication without re-write to clarify several aspects, mainly:~~
- clarifying the statement of the problem "why Indonesia in specific"? Accepted modification
- clarifying the procedures of validations of the instrument "the test". Still a concern! But not a big problem
- clarifying the procedures of selecting the research sample "is it random? and how?" still a concern, but OK
- clarifying the original meanings "the full name at first use" all of the abbreviations, like: MA Darun, RI, FA, MN, in context, and:DN, RD, NA, ES, SY, RI, DR, PR, UN, AK, DI, MF, RM, ET, CN, NL, MN, MA, etc in tables". Good modification
- clarifying the basis of the data representations in all tables "the classifications based on what?" OK
- rewriting the whole paper by expert in English. Good modification

Resources: • Some of it outdated, besides the need to support the claims of: "especially for students in Indonesia" and " especially in Indonesia", by adding research from Indonesia. OK • Need to be more specific, where there are a number of resources for a very limited cited words or results wuth no real needs. OK
11. Please indicate the strong aspects of the research that is reported.
• There is a good effort in representing the findings. • The overall paper is good in representing the whole work "aim, procedures, and results" after real modifications and clarifications.
12. Please indicate the weak aspects of the research reported.
As mentioned in General Comments: • The paper Cannot be accepted for publication without re-write to clarify several aspects, mainly: OK ▪ ~~clarifying the statement of the problem "why Indonesia in specific"?~~ OK ▪ clarifying the procedures of validations of the instrument "the test". Still a weekness, but ok ▪ clarifying the procedures of selecting the research sample "is it random? and how?" Still not clear, but ok ▪ ~~clarifying the original meanings "the full name at first use" all of the abbreviations.~~ Good ▪ ~~clarifying the basis of the data representations in all tables "the classifications based on what?"~~ OK ▪ ~~rewriting the whole paper by expert in English:~~ Good with some mistakes “mentioned on the paper” – ~~for example: avoid exessive use of a spesific terms, like "as well as" 5 times,~~ Good – ~~dealing with the use of the past tenses and the present progressive, etc.,~~ Good
13. Final recommendation:

Can be published	✓
Can be published provided that the suggested amendments are made	
Must be amended and resubmitted for evaluation	~~✓~~
Should be rejected	

14. Comments: Please indicate in the space below any comments and suggestions for improving the article. ~~If researcher/researchers are able to modify the papaer to deal with the weekness mentioned upove, it might be possible to accept the paper for publication due to the good represenations of the findings, otherwise it should be rejected.~~ OK, but need to correct the missbiings on the paper in few words “mentioned on the paper”.

17 November 2017

EXPLORING OF MULTI MATHEMATICAL REPRESENTATION CAPABILITY IN PROBLEM SOLVING: STUDY ON SENIOR HIGH SCHOOL STUDENTS IN INDONESIA

2_SAL-sa_review

Abstract

This article discusses the students' multi-mathematical representation abilities in problem solving, especially on the linear equation system of two variables material. Data were collected from 48 students using written tests and in-depth interviews on selected subjects. The research findings showed that 8.33% of students solved problem using three representations namely symbolic - verbal - table representation, 37.5% of students solved problem using three representations namely symbolic - verbal - images representation, 45.83% of students solved problem using two representations namely symbolic – verbal representation, and the rest using symbolic representation. In the use of verbal representation, there were still students who had difficulty in composing words, and all students had difficulties on the translational process from symbolic representation as well as verbal representation to other representations. Ability to understand concepts and relationships between mathematical concepts was a necessary condition for the achievement of multi mathematical representation capabilities. It is therefore recommended that teachers use many representations such as verbal tables, images to enhance students' understanding on the material.

Key words: *multiple representation; problem solving; linear equation system of two variables.*

Introduction

Problem-solving abilities are related to metacognition (Muhtarom, Juniati & Siswono, 2017), as well as mathematical representation abilities. Therefore, mathematical representation ability is important in understanding mathematics because students face mathematical problems to be solved as well as when they have to communicate their solutions to others and it is a form of attitude in mathematics. Representations can be represented in representations of images, symbols, and signs (notations) which are forms of mathematical concepts (Vergnaud, 1997). Representation is a kind of configuration process to present something in other situations involving identification, selection and ideas delivery (Goldin & Kaput, 1996; Romberg & Kaput, 1999; Seeger et al, 1998). Everything made by students to internalize and show their work is called representation (Kalathil & Sherin, 2013). Even Hwang, Chen, Dung, & Yang (2007) conclude that good representational skills are the key to gain the right solutions to solve problems.

Each problem can be solved by presenting different representations (Gagatsis & Elia, 2004) so that the issues and their representations in this case are closely related. Mathematical representations associated with different ways such as symbols, diagrams, tables, verbal statements and figures and others at the same time are called multiple-representation (Brenner et al., 1997; Nizaruddin & Waluyo, 2015). The use of multiple representations allows one to explore mathematical concepts in various ways, and emphasizes that not necessarily only one mathematical process produces problem solutions (Romberg & Kaput, 1999). Keller & Hirsch (1998) emphasize that the use of multiple representations is as an advantage, since many representations avoid the limitation of one type of representation and build something new, which is clearer and more useful for the problem-solving process. Thus, multiple representations has many functions, namely 1) completing each other between one

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representation and another representation; 2) limiting the interpretation on the other representations; 3) supporting the process of constructive understanding (Ainsworth, 2006). It means that the use of multiple representations is recommended by many mathematical educators. Similarly, support from the community of mathematics education states that students can interpret the concept of mathematics through the experience of multiple-representation (Janvier, 1987; Sierpinska, 1992).

Using different representations can make the concepts more accessible and enjoyable for learners. Romberg & Kaput (1999) and Hiebert & Carpenter (1992) state that the use of multiple representation helps students learn by using their own thinking and learning habits. Nizaruddin et al, (2017) provides the view that game-based learning can improve students' problem-solving skills, and furthermore it is expected to develop the representation skills used in solving math problems. It can be realized if learning facilitates students in accommodation thinking to develop their own knowledge schemes (Muhtarom, Yanuar & Sutrisno, 2017). Therefore learning process that uses two representations is better than one representation (Bransford & Schwartz, 1999; Ainsworth, 2006). Therefore, an understanding of multiple representation can support students in understanding every mathematical concept learned, and can anticipate the fallacy of concepts in mathematics. This means the ability to understand concepts and relationships between mathematics concepts is a necessary condition for the achievement of multi mathematical representation capabilities.

In Indonesia, the study of multiple-representation capabilities is rarely discussed; if there is usually only a discussion of the representation used by students (Santia, 2015; Tyas, 2016; Muhamad, 2017) and does not provide the view on the multiple-representation ability of students as well as the translation process from one representation to another. Therefore, it is interesting to do a study on students' multi mathematical representation in mathematical problem solving, especially for students in Indonesia. The purpose of this study is to determine the ability of multi-mathematical representation of students in solving mathematical problems in the material system of two linear equations.

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What is the research question/problem that you want to explore?

Methodology of Research

General Background of Research

This research was qualitative exploring multi mathematical representation abilities in problem solving especially on linear equation system of two variables. The results of this study will be the basis for developing learning tools on linear equations system of two variables based on multiple representations so as to improve students' creative thinking ability, especially in Indonesia.

Sample of Research

Subjects in this study were 48 Senior High School students of MA Darun Najah Margoyoso District of Pati Regency academic year 2017/2018. Selection of subjects was based on the consideration: 1) students already had got enough material so that they were expected to be able to complete a math problem on linear equation system of two variables, 2) subjects were selected considering having good communication skills to be able to express the solving process of resolving, 3) in problem solving, they used some kinds of representations.

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Instrument and Procedures

The instrument of this research is a written test of problem solving and interview guides based on problem solving tasks to explore the multiple-representation ability of students. Prior to use the instrument was validated first by three validators who are experts in mathematics education, so declared eligible to be used to take research data. The written test instrument is as follows: "A two-variable linear equation system has a finite set {3.3}. If the first variable coefficient is an even number and the second variable coefficient is an odd number, make a system example linear equation of two variables! Also show me the steps to solve it by using some representations which you master.

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Data Analysis

The first stage in the data collection, students were asked to solve mathematical problems related to material of linear equation system of two variables by using various representations that were mastered. Based on the data, data analysis and reduction were done to obtain detailed description of students' multi mathematical representation abilities in problem solving. Next, four suitable subjects who fulfilled the criteria were selected to do semi-structured interviews focused on matters relating to the multiple-representation abilities of students in solving the problems of linear equation system of two variables. The four subjects were RI who used three representations namely symbolic - verbal - image representations; FA who used three representations namely symbolic - verbal - tables, DP who used two representations namely symbolic - verbal, and MN subjects that used only symbolic representation in solving math problems (see table 1). Next, data were analyzed and validated using triangulation method, where data of triangulation result was valid subject data to give description of abilities of students' multi mathematical representation.

Results of Research

Table 1 was very clear in describing the results of data analysis conducted on 48 students. We can know the type of representation used by students, namely: symbolic - verbal - images representations, symbolic - verbal - table representations, symbolic - verbal representations, and symbolic representation to solve problems. The percentage of students using three representations namely symbolic - verbal - table representation was 8.33%, using symbolic - verbal - images representation was 37.5% while those using two representations namely symbolic – verbal representations was 45.83% and those using only one representation was 8.33%..

Table 1. Representation used by subjects.

Representation	Type of representation	Subject	%	Selected subject*
three representations	symbolic - verbal - images	DN, RD, NA, ES, SY, RI*, DR, PR, UN, AK, DI, MF, RM, ET, CN, NL, MN, MA	37.5%	RI
	symbolic - verbal - table	FA*, MK, NN, BR	8.33%	FA
two representations	symbolic - verbal	SW, ZN, NS, K, IA, RW, NDS, DP*, NM, CL, RM,	45.83%	DP

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Representation	Type of representation	Subject	%	Selected subject*
		ZY, AP, PN, HN, ZN, PL, MD, EF, NS, IK, VN		
one representation	symbolic	ME, MN*, ND, AS	8.33%	MN

Subject RI solved the problem of linear equation system of two-variables by using 3 representations namely symbolic - verbal - image representations. When answering using symbolic and verbal representations, subject RI could answer correctly but when answering by using image representation there were mistakes in the answer. Clearly, representation and thinking process of RI's problem solving is presented in Table 2.

Table 2. Analysis result of subject RI.

Thinking Process	Representation		
	Symbolic	**Verbal**	**Image**
Procedure oriented	Capable to manipulate symbols in result completion completely and systematically.	Able to use the right procedure.	Has not been able to use the proper procedure in obtaining the image form of the problem.
Process oriented	Able to interpret the meaning of the symbols written.	Able to apply the process well even though there is little mistake.	Has not understood and has not been able to apply the problem solving process well.
Math object-oriented	Able to operate the relevant mathematical symbols.	Able to use words in understanding the mathematical objects.	Has not been able to interpret images in problem solving.
Concept oriented	Able to relate procedures and processes applied to symbolic representations in mathematical concepts and able to identify concepts used in mathematical operations.	Able to relate procedures and processes applied to verbal representation in mathematical concepts and able to identify concepts used in mathematical operations.	Has not been able to relate the procedures and processes applied to image representation in mathematical concepts and able to identify concepts used in mathematical operations.

Subject FA solved the problem of a two-variable linear equation system using 3 representations namely symbolic - verbal - image representations. When answering by using symbolic representation, subject FA could answer correctly but when answering by using verbal and table representations, there were some mistakes in the answer. To make it clearer, the representation and thinking process of subject FA's problem solving are presented in Table 3.

Table 3. Analysis result of subject FA.

Thinking Process	Representation		
	Symbolic	**Verbal**	**Table**
Procedure oriented	Able to manipulate symbols in complete and systematic completion results.	Has not been able to use the procedure appropriately.	Has not been able to use the proper procedure in obtaining the completion result

Thinking Process	Representation		
	Symbolic	Verbal	Table
Process oriented	Able to interpret the meaning of the symbols written.	Has not been able to implement the problem-solving process properly.	Has not understood and implemented problem solving process
Math object oriented	Able to operate the relevant mathematical symbols.	Able to use words in understanding the mathematical objects.	Has not been able to interpret the problem in the form of mathematical tables.
Concept oriented	Able to relate procedures and processes applied to symbolic representations in mathematical concepts and able to identify concepts used in mathematical operations.	Has not been able to relate the procedures and processes applied to verbal representation in mathematical concepts and able to identify concepts used in mathematical operations.	Has not been able to relate procedures and processes applied to precise table representation and has not been able to identify and operate mathematical concepts.

Subject DP solved the problem of a two-variable linear equation system using two representations namely symbolic-verbal representations. When answering by using symbolic representation it appeared that subject was able to use the relevant symbols namely variables x and y, able to pass the completion process well even though the procedure used was not yet complete. Subject was able to interpret the symbols used and able to relate procedures and processes applied on symbolic representations in mathematical concepts and able to identify concepts used in mathematics operations. Subject DP could answer correctly and realized the mistakes in the completion process using verbal representation because of his understanding on the characteristics of two variables linear equations system. Clearly, the representation and subject DP's problem-solving thinking process are presented in Table 4.

Table 4. Analysis result of subject DP.

Thinking Process	Representation	
	Symbolic	Verbal
Procedure oriented	Able to manipulate symbols in his work result although it is not complete yet.	Able to use appropriate and systematic procedures
Process oriented	Able interpret the meaning of the symbols written.	Able to apply the process correctly.
Math object-oriented	Able to operate the relevant mathematical symbols.	Able to use words in the understanding of the mathematical object used
Concept oriented	Able to relate procedures and processes applied on symbolic representations in mathematics concepts and able to identify concepts used in mathematics operations.	Able to relate procedures and processes applied to verbal representation in mathematical concepts and able to identify concepts used in mathematics operations.

Subject MN solved the problem of a two-variable linear equation system using symbolic representation. It was seen from the use of variables x and y. Subject MN had not been able to use good procedures and processes because there were still many verbal representations. Basically the subject had been able to interpret the symbol used but he had not been able to relate the procedures and processes applied on the symbolic representation in mathematics concepts and able to identify the concepts used in mathematics operations.

Clearly, the representation and the problem-solving thinking process of subject DP are presented in Table 5.

Table 5. Analysis result of subject MN.

Thinking Process	Symbolic Representation
Procedure oriented	Has not been able to manipulate symbols in completion result completely and systematically.
Process oriented	Able to interpret the meaning of the symbols written.
Mathematics object oriented	Able to operate with relevant symbols.
Concept oriented	Has not yet been able to relate to the processes and procedures applied on the symbolic representation in mathematics concept and able to identify the concepts used in mathematics operations.

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Discussion

The results of this study provided the view that students basically had the ability to solve problems in various representations. It was seen only 8.33% of students who solved problems in one type of symbolic representation only. It showed that students who had high independence could show various representations, unlike other students who only showed one representation in solving the problem (Nizaruddin & Waluyo, 2015). In addition, students' understanding of mathematics concepts was reflected in many representations that students use to solve problems (Brenner et al., 1997). However, we had to be aware that there were still students who had difficulty in the process of transferring translation of representation use for example in the beginning they used symbol representation, then using verbal representation and image (other representation). It was in line with the research of Gagatsis & Elia (2004) which concluded that students had many difficulties in changing the representation of symbols (algebra) into verbal representations. Investigation Janvier (1987) showed how the achievement of transfer between one representation and the other representation varied depending on the nature of the relationship between the chosen representations. Thus to be able to make different representations, knowledge of the subject matter as well as knowledge representation were required because not all the material could be presented in every representation.

It provided an overview of the importance of understanding ability in concepts and relationships between mathematics concepts which were the main requirements in achieving multi mathematical representation abilities. The fact that students were incapable of creating a proper representation in understanding the problem was thought because teachers, especially in Indonesia often only used symbolic representation in solving mathematical problems, including the material of two-variable linear equation system. In fact, we realized that using different representations could make the concepts more accessible and enjoyable for learners, so solving the same problems using different representations or multiple representations showed better results than one representation (Hiebert & Carpenter (1992; Romberg & Kaput, 1999). Even (Keller & Hirsch, 1998; Bransford & Schwartz, 1999; Ainsworth, 2006) state that two representations are better than one representation because they avoid the limitations of one type of representation and build a new, clearer representation for the problem-solving process. It meant that the ability to understand concepts and relationships between mathematical concepts was a necessary condition for the achievement of multi mathematical representation capabilities, and multiple-representation ability itself could support students in

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understanding every mathematical concept and could anticipate mistakes in understanding mathematical concepts.

Conclusions

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Bring out the significance of your research. Show how you've brought closure to the research problem, and point out remaining gaps in knowledge by suggesting issues for further research

Students tend to use symbolic representation rather than other representations in solving problems of two-variables linear equations system. When students use verbal representation they tend to have difficulty in composing words whereas in image or table representation they have not been able to solve the problem correctly. Based on the facts, we know that the selection of representations used greatly affects the students' answers because in using the representation it takes knowledge of the mathematical concepts as well as the understanding of the representation itself. We hope this article can be further developed to get an overview of students' multi-representation abilities on other materials so that teachers can design learning that is able to develop the ability.

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Referencing needs to be brought in line with the APA system

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The manuscript is in **dire need** of proofreading to improve its English

TAMBAHAN BAHAN BAB METODE

Uji coba terhadap instrument penelitian dilaksanakan pada tanggal 30 Mei 2017. Dari hasil uji coba tersebut terlihat bahwa soal tersebut dapat diselesaikan dengan berbagai representasi. Hasil uji coba kemudian dianalisis berdasarkan validitas dan reliabilitas untuk mengetahui kelayakan soal sebagai instrumen penelitian.

Penelitian ini mengambil subjek penelitian siswa kelas XI. Tahap awalnya siswa diberikan tes tertulis. Dalam penelitian ini, tes tertulis diikuti oleh 48 calon subjek terdiri dari 5 siswa laki – laki orang dan 43 siswa perempuan. Penentuan subjek penelitian juga ditinjau dari perolehan nilai rapot matematika siswa.

Pengambilan data dalam penelitian ini silakukan dua tahap. Tahap Pertama adalah memberikan instrumen tes tertulis. Instrumen tes tertulis diujikan pada tanggal 14 Juli 2014 Pukul 07.00 WIB diruang kelas XII IPA dan pukul 08.40 diruang kelas XI IPS B. Dilanjutkan tahap kedua adalah melaksanakan kegiatan wawancara terhadap subjek yang terpilih. Wawancara dilaksanakan berdasarkan kesepakatan antara peneliti dengan sunjek penelitian. Pelaksanaan wawancara secara rinci disajkan pada tabel beriku:

Daftar Pelaksanaan Wawancara

No	Tanggal	Inisial	Waktu	Lokasi
1	28 Juli 2017	DP	12.10	Sekolah
2	28 Juli 2017	FA	12.55	Sekolah
3	31 Juli 2017	MN	10.00	Sekolah
4	31 Juli 2017	RI	10.40	Sekolah

Seluruh rangkaian kegiatan pengambilan data penelitian dari ujian tes tertulis hingga wawancara telah didokumentasikan. Tujuan dari dokumentasi ini sendiri adalah agar data yang dihasilkan dalam penelitian tidak diragukan.

EXPLORING OF MULTI MATHEMATICAL REPRESENTATION CAPABILITY IN PROBLEM SOLVING ON SENIOR HIGH SCHOOL STUDENTS

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Reworked

Abstract

The students' multi-mathematical representation capability in problem solving is very important and interesting to discuss, specifically for problems in the two-variable linear equation system. Data was collected from 48 students using written tests and in-depth interviews with selected participants. The research findings showed that few students are using three representations namely symbolic - verbal - table representation, and symbolic representation, however most of the students are using three representations namely symbolic - verbal - images representation, and two representations namely symbolic – verbal representations, and the rest used symbolic representation. In the use of verbal representation, some students had difficulty composing words and all students encountered difficulties in the translational process from symbolic representation and verbal representation to other types of representation. The ability to understand concepts and relationships between mathematical concepts was found to be a necessary condition for the achievement of multi-mathematical representation capability. It is therefore recommended that teachers use a variety of different types of representation, such as verbal, tables and images, to enhance students' understanding of the material.

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Key words: *multiple representations; problem solving; two-variable linear equation system.*

Introduction

Mathematical representation ability is important in understanding mathematics, both when students are solving mathematical problems and when they need to communicate their solutions to others, and it is also a form of attitude in mathematics. Problem-solving abilities are related to metacognition (Muhtarom, Juniati & Siswono, 2017), as well as mathematical representation abilities. Moreover, representation can be presented in the form of images, symbols, and signs (notation), which are all forms of mathematical concepts (Vergnaud, 1997). Representation is a kind of configuration process to present something in a different situation which involves identification, selection and the delivery of ideas (Goldin & Kaput, 1996; Romberg & Kaput, 1999; Seeger et al, 1998). Everything that students do to externalize and show their work is called representation (Kalathil & Sherin, 2013). Hwang, Chen, Dung, & Yang (2007) conclude that good representational skills are the key to gaining the right solutions to solve problems.

Every problem can be solved by using different types of representation (Gagatsis & Elia, 2004) and this means there is a close relationship between the problem and its representations. Mathematical representations that are associated with the simultaneous use of different methods, such as symbols, diagrams, tables, verbal statements, figures and so on, are known as multiple-representations (Brenner et al., 1997). The use of multiple representations allows one to explore mathematical concepts in various ways, and emphasizes that there may be more than one possible mathematical process for producing a solution to a problem (Romberg & Kaput, 1999). Keller & Hirsch (1998) stress that the use of multiple representations is an advantage, since it avoids limitation to a single type of representation and builds something new, which is clearer and more useful for the problem-solving process. Thus, there are several functions of multiple representation: 1) the different representations complement each other; 2) the interpretation of other representations is limited; 3) the process of constructive understanding is enhanced (Ainsworth, 2006). This means that the use of multiple representations is recommended by many mathematical educators. Similarly, the mathematics community supports the idea that students can interpret mathematical concepts through the experience of multiple representations (Janvier, 1987; Sierpinska, 1992).

The use of different representations can make concepts more accessible and enjoyable for the student. Romberg & Kaput (1999) and Hiebert & Carpenter (1992) state that the use of multiple representations helps students to learn by using their own thinking and learning habits. Nizaruddin et al, (2017) offers the view that game-based learning can improve students' problem-solving skills, and furthermore it has the potential to develop the representation skills needed for solving mathematics problems. It is generally recognized that learning facilitates students in accommodating their thoughts to develop their own knowledge schemes (Muhtarom, Yanuar & Sutrisno, 2017). Therefore, a learning process that uses two representations is better than a process using only a single representation (Bransford & Schwartz, 1999; Ainsworth, 2006). Thus, an understanding of multiple representations can support students' comprehension of mathematical concepts, as well as anticipating the fallacy of concepts in mathematics. This means that the ability to understand concepts and relationships between mathematical concepts is a necessary condition for the achievement of multi-mathematical representation capability.

In Indonesia, to date there has been little research on multiple-representation capability; existing studies tend to discuss only the types of representation used by students (Santia, 2015; Tyas, 2016; Muhamad, 2017) and do not offer views on the multiple-representation capability of students or the translation process from one representation to another. Therefore, it is interesting to research students' multi-mathematical representation in problem solving, especially among students in Indonesia. The purpose of this current research is to determine the multi-mathematical representation capability of students in solving mathematical problems, specifically two-variable linear equation problems.

Research on multiple representations is an important topic to address because the identification of multiple representation capability can be used for developing teaching material. In addition, the importance of multiple representations in this research can be used to improve students' creative thinking. This research will explore multiple representation capability, specifically in mathematical problem solving in two-variable linear equation problems, observing the following thought processes: procedure-oriented, process-oriented, math object-oriented and concept-oriented.

Commented [S2]: Or representations?

Commented [S3]: capability

Commented [S4]: Or representations?

Research Methodology

General Background of Research

This research is qualitative and explores multi-mathematical representation capability in problem solving, specifically in two-variable linear equation problems. The results of the research can be used as the basis for developing teaching material for the two-variable linear equation system based on multiple representations so as to improve students' creative thinking ability, especially in Indonesia.

Sample of Research

The participants in this research were 48 Senior High School students from the Madrasah Aliyah Darun Najah Senior High School in the Margoyoso District of Pati Regency during the academic year 2017/2018. The selection of participants was based on the following considerations: 1) the students had already been given sufficient material that they were expected to be able to complete a two-variable linear equation mathematics problem, 2) the participants selected were considered to possess good communication skills enabling them to express the process used for solving the problem, 3) in problem solving, the participants used a number of different kinds of representation. A trial of the research instrument was conducted on May 30, 2017. From the results of the trial, it was seen that the problem could be solved using various types of representation. The test results were then analyzed based on validity and reliability to determine the feasibility of the problem as a research instrument. This research involved participants who were class XI students. In the initial stage, the students were given a written test. In this research, the written test was taken by 48 prospective participants consisting of 5 male students and 43 female students. The selection of research subjects was also based on a review of the students' mathematics grades.

Instrument and Procedures

The instrument used in this research was a written test on problem solving and a guided interview based on the problem solving task to explore the multiple-representation capability of students. Prior to use, the instrument was first validated by three validators who are experts in mathematics education, and was declared eligible to be used for obtaining research data. The written test instrument was as follows: "A two-variable linear equation system has a finite set {3.3}. If the first variable coefficient is an even number and the second variable coefficient is an odd number, make a two-variable linear equation example! Also show the steps used to solve the problem using your own choice of representations”. The data was collected in two stages. The first stage was the written test, as described above, which was held on July 14, 2017 at 7 a.m. in the classroom of XI IPA and at 8.40 a.m. in the classroom of XI IPS. The second phase was to conduct interviews with selected subjects. The interviews were conducted based on an agreement between the researcher and the research scientist. The details of the interviews are presented in Table 1 below:

Table 1 List of Interviews

No	Date	Participant’s Code	Time	Location
1	28 July 2017	DP	1 p.m.	School
2	28 July 2017	FA	2 p.m.	School
3	31 July 2017	MN	10 a.m.	School
4	31 July 2017	RI	11 a.m.	School

Data Analysis

In the first stage of the data collection, students were asked to solve a two-variable linear equation problem using various representations that they were familiar with. Based on the data obtained from this test, a data analysis and reduction was then performed to obtain a detailed description of the students' multi-mathematical representation capabilities in problem solving. Next, four suitable participants who fulfilled the necessary criteria were selected to participate in semi-structured interviews which focused on their multiple-representation capabilities in solving a two-variable linear equation problem. The four participants were RI (participant with code RI) who used three representations, namely symbolic - verbal - image representations; FA (participant with code FA) who used three representations, namely symbolic - verbal - tables, DP who used two representations, namely symbolic - verbal, and MN (participant with code MN) who used only symbolic representation in solving the mathematics problem (see Table 1). Next, the data was analyzed and validated using the triangulation method, where data from the results of the triangulation was valid participant data for providing a description of the students' multi-mathematical representation capabilities.

Results of Research

Table 2 presents a clear description of the results of the data analysis of the 48 students. From this, the types of representation used by students for solving the problem can be identified, namely: symbolic - verbal - image representations, symbolic - verbal - table representations, symbolic - verbal representations, and symbolic representation. The percentage of students using three representations, namely symbolic - verbal - table representation was 8.33%, the use of symbolic - verbal - image representation was 37.5%, those using two representations namely symbolic – verbal representations was 45.83% and those using only one representation was 8.33%.

Table 2. Representation used by participants.

Representation	Type of representation	Participant's Code	%	Selected participants with code*
three representations	symbolic - verbal - images	DN, RD, NA, ES, SY, RI*, DR, PR, UN, AK, DI, MF, RM, ET, CN, NL, MN, MA	37.5	RI
	symbolic - verbal - table	FA*, MK, NN, BR	8.33	FA
two representations	symbolic - verbal	SW, ZN, NS, K, IA, RW, NDS, DP*, NM, CL, RM, ZY, AP, PN, HN, ZN, PL, MD, EF, NS, IK, VN	45.83	DP
one representation	symbolic	ME, MN*, ND, AS	8.33	MN

Participant RI solved the two-variable linear equation problem using 3 representations, namely symbolic - verbal - image representations. When answering using symbolic and verbal representations, participant RI was able to answer correctly but when answering using image representation there were mistakes in the answer. The representation and thought process of RI's problem solving is shown clearly in Table 3.

Table 3. Analysis of results of participant with code RI.

Thought Process	Representation		
	Symbolic	Verbal	Image
Procedure-oriented	Able to manipulate symbols to complete the results comprehensively and systematically.	Able to use the right procedure.	Unable to use the proper procedure to obtain the image form of the problem.
Process-oriented	Able to interpret the meaning of written	Able to apply the process well with few	Unable to understand or apply the problem-

	symbols.	mistakes.	solving process properly.
Math object-oriented	Able to operate the relevant mathematical symbols.	Able to use words in understanding the mathematical objects.	Unable to interpret images in problem solving.
Concept-oriented	Able to relate procedures and processes applied to symbolic representation in mathematical concepts and able to identify concepts used in mathematical operations.	Able to relate procedures and processes applied to verbal representation in mathematical concepts and able to identify concepts used in mathematical operations.	Unable to relate procedures and processes applied to image representation in mathematical concepts but able to identify concepts used in mathematical operations.

The participant with the code FA solved the two-variable linear equation problem using 3 representations, namely symbolic - verbal - image representations. When answering with the use of symbolic representation, participant FA was able to answer correctly but when answering using verbal and table representations, there were a number of mistakes in the answer. For a clearer picture, the representation and thought process of participant FA's problem solving are presented in Table 4.

Table 4. Analysis of results of participant with code FA.

Thought Process	Representation		
	Symbolic	**Verbal**	**Table**
Procedure-oriented	Able to manipulate symbols to complete the results comprehensively and systematically.	Unable to use the procedure appropriately.	Unable to use the proper procedure to complete the results.
Process-oriented	Able to interpret the meaning of written symbols.	Unable to implement the problem-solving process properly.	Unable to understand and implement the problem solving process.
Math object-oriented	Able to operate the relevant mathematical symbols.	Able to use words in understanding the mathematical objects.	Unable to interpret the problem in the form of mathematical tables.
Concept-oriented	Able to relate procedures and processes applied to symbolic representation in mathematical concepts and able to identify concepts used in mathematical operations.	Unable to relate procedures and processes applied to verbal representation in mathematical concepts but able to identify concepts used in mathematical operations.	Unable to relate procedures and processes applied to precise table representation and unable to identify and operate mathematical concepts.

The participant with code DP solved the two-variable linear equation problem using two representations, namely symbolic - verbal representations. When answering with the use of symbolic representation, it appeared that the participant was able to use the relevant symbols, namely variables x and y, and able to complete the process well even though the procedure used was not yet complete. The participant was able to interpret the symbols used, able to relate the procedures and processes applied to symbolic representation in the mathematical concepts, and able to identify the concepts used in the mathematical operations. Participant DP was able to answer correctly and was aware of the mistakes made in the completion process using verbal representation because of his understanding of the characteristics of the two-variable linear equation system. The representation and thought process of participant DP's problem-solving are presented in Table 5.

Table 5. Analysis of results of participant with code DP.

Thought Process	Representation	
	Symbolic	Verbal
Procedure-oriented	Able to manipulate symbols in the results although still incomplete.	Able to use appropriate and systematic procedures.
Process-oriented	Able to interpret the meaning of written symbols.	Able to apply the process correctly.
Math object-oriented	Able to operate the relevant mathematical symbols.	Able to use words in understanding the mathematical objects used.
Concept-oriented	Able to relate procedures and processes applied to symbolic representation in mathematical concepts and able to identify concepts used in mathematical operations.	Able to relate procedures and processes applied to verbal representation in mathematical concepts and able to identify concepts used in mathematical operations.

Table 6. Analysis of results of participant with code MN.

Thought Process	Symbolic Representation
Procedure-oriented	Unable to manipulate symbols to complete the results comprehensively and systematically.
Process-oriented	Able to interpret the meaning of written symbols.
Math object-oriented	Able to operate the relevant mathematical symbols.
Concept-oriented	Unable to relate processes and procedures applied to symbolic representation in mathematical concepts but able to identify concepts used in mathematical operations.

The participant with the code MN solved the problem of a two-variable linear equation system using symbolic representation. This is evident from the use of variables x and y. Participant MN was unable to use the proper procedures and processes because there was still a large amount of verbal representation. The participant was essentially able to interpret the symbols used but was unable to relate the procedures and processes applied to the symbolic representation in mathematical concepts though able to identify the concepts used in mathematical operations. The representation and the problem-solving thought process of the participant with code DP are presented in Table 6.

Discussion

The results of this research show that students essentially have the ability to solve problems using various types of representation. It was found that only 8.33% of students solved the problem presented using only a single type of symbolic representation. Students who had a high level of independence were seen to use a number of different types of representation, unlike other students who used only a single type of representation to solve the problem. In addition, the students' understanding of mathematical concepts was reflected in the greater number of representations used to solve the problem (Brenner et al., 1997). However, it should be noted that some of the students still encountered difficulties in the process of transferring the use of different types of representation, for example, in the beginning they used symbolic representation, then verbal or image representation (other types of representation). This is in line with the research of Gagatsis & Elia (2004) which concludes that students encounter numerous difficulties in changing from symbolic representation (algebra) into verbal representation. An investigation by Janvier (1987) shows how a successful transfer between one type of representation and another varies depending on the nature of the relationship between the chosen representations. Thus, in order to use different representations, students require knowledge of the subject matter as well as knowledge about representation because not all material can be presented using all types of representation.

This provides an overview of the importance of the ability to understand concepts and relationships between mathematical concepts, which are the main requirements for achieving multi-mathematical representation capability. The fact that some students were incapable of creating a proper representation for understanding the problem is believed to be because teachers, especially in Indonesia, often use only symbolic representation for solving mathematical problems, including two-variable linear equation problems. It should be recognized that the use of different representations makes concepts more accessible and enjoyable for students, and for this reason, solving the same problem using different representations or multiple representations shows better results than the use of a single representation (Hiebert & Carpenter 1992; Romberg & Kaput, 1999). Ainsworth (2006); Bransford & Schwartz (1999); and Keller & Hirsch (1998) state that two representations are better than one because they avoid the limitations of a single type of representation and build a new, clearer representation for the problem-solving process. This means that the ability to understand concepts and relationships between mathematical concepts is a necessary condition for the achievement of multi-mathematical representation capability, and multiple-representation ability itself is able to support students in understanding each different mathematical concept and anticipating mistakes in understanding mathematical concepts.

Conclusions

Students tend towards the use of symbolic representation rather than other types of representation in solving two-variable linear equation problems. When students use verbal representation, they are inclined to have difficulty in composing words while in image or table representation they are unable to solve the problem correctly. Based on these facts, we can conclude that the choice of representation used to solve a problem greatly affects the students' answers because the use of a particular type of representation requires knowledge of the mathematical concept as well as an understanding of the representation itself. Suggestions for further research include an exploration of the students' thought process in transitioning between different types of representation, such as verbal representation to symbolic representation or vice versa.

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EXPLORING OF MULTI MATHEMATICAL REPRESENTATION CAPABILITY IN PROBLEM SOLVING: STUDY ON SENIOR HIGH SCHOOL STUDENTS IN INDONESIA

Nizaruddin
Mathematics Education Department, Universitas PGRI Semarang, Indonesia
E-mail: nizaruddin@upgris.ac.id

Muhtarom
Mathematics Education Department, Universitas PGRI Semarang, Indonesia
E-mail: muhtarom@upgris.ac.id

Yanuar Hery Murtianto
Mathematics Education Department, Universitas PGRI Semarang, Indonesia
E-mail: yanuarheri@upgris.ac.id

Abstract

The students' multi-mathematical representation capability in problem solving are very important and interesting to discuss, specifically for problems in the two-variable linear equation system. Data was collected from 48 students using written tests and in-depth interviews with selected participants. The research findings showed that few students are using three representations namely symbolic - verbal - table representation, and symbolic representation, however most of the students are using three representations namely symbolic - verbal - images representation, and two representations namely symbolic – verbal representations, and the rest used symbolic representation. In the use of verbal representation, some students had difficulty composing words and all students encountered difficulties in the translational process from symbolic representation and verbal representation to other types of representation. The ability to understand concepts and relationships between mathematical concepts was found to be a necessary condition for the achievement of multi-mathematical representation capability. It is therefore recommended that teachers use a variety of different types of representation, such as verbal, tables and images, to enhance students' understanding of the material.
Key words: *multiple representations; problem solving; two-variable linear equation system.*

Introduction

Mathematical representation ability is important in understanding mathematics, both when students are solving mathematical problems and when they need to communicate their solutions to others, and it is also a form of attitude in mathematics. Problem-solving abilities are related to metacognition (Muhtarom, Juniati & Siswono, 2017), as well as mathematical representation abilities. Moreover, representation can be presented in the form of images, symbols, and signs (notation), which are all forms of mathematical concepts (Vergnaud, 1997). Representation is a kind of configuration process to present something in a different situation which involves identification, selection and the delivery of ideas (Goldin & Kaput, 1996; Romberg & Kaput, 1999; Seeger et al, 1998). Everything that students do to externalize and show their work is called representation (Kalathil & Sherin, 2013). Hwang, Chen, Dung, & Yang (2007) conclude that good representational skills are the key to gaining the right solutions to solve problems.

Every problem can be solved by using different types of representation (Gagatsis & Elia, 2004) and this means there is a close relationship between the problem and its representations. Mathematical representations that are associated with the simultaneous use of different methods, such as symbols, diagrams, tables, verbal statements, figures and so on, are known as multiple-representations (Brenner et al., 1997). The use of multiple representations allows one to explore mathematical concepts in various ways, and emphasizes that there may be more than one possible mathematical process for producing a solution to a problem (Romberg & Kaput, 1999). Keller & Hirsch (1998) stress that the use of multiple representations is an advantage, since it avoids limitation to a single type of representation and builds something new, which is clearer and more useful for the problem-solving process. Thus, there are several functions of multiple representation: 1) the different representations complement each other; 2) the interpretation of other representations is limited; 3) the process of constructive understanding is enhanced (Ainsworth, 2006). This means that the use of multiple representations is recommended by many mathematical educators. Similarly, the mathematics community supports the idea that students can interpret mathematical concepts through the experience of multiple representations (Janvier, 1987; Sierpinska, 1992).

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Research Methodology

General Background of Research

This research is qualitative and explores multi-mathematical representation capability in problem solving, specifically in two-variable linear equation problems. The results of the research can be used as the basis for developing teaching material for the two-variable linear equation system based on multiple representations so as to improve students' creative thinking ability, especially in Indonesia.

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	symbolic - verbal - table	FA*, MK, NN, BR	8.33	FA
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Thought Process	Representation		
	Symbolic	Verbal	Image
Procedure-oriented	Able to manipulate symbols to complete the results comprehensively and systematically.	Able to use the right procedure.	Unable to use the proper procedure to obtain the image form of the problem.
Process-oriented	Able to interpret the meaning of written symbols.	Able to apply the process well with few mistakes.	Unable to understand or apply the problem-solving process properly.
Math object-oriented	Able to operate the relevant mathematical symbols.	Able to use words in understanding the mathematical objects.	Unable to interpret images in problem solving.
Concept-oriented	Able to relate procedures and processes applied to symbolic representation in mathematical concepts and able to identify concepts used in mathematical operations.	Able to relate procedures and processes applied to verbal representation in mathematical concepts and able to identify concepts used in mathematical operations.	Unable to relate procedures and processes applied to image representation in mathematical concepts but able to identify concepts used in mathematical operations.

The participant with the code FA solved the two-variable linear equation problem using 3 representations, namely symbolic - verbal - image representations. When answering with the use of symbolic representation, participant FA was able to answer correctly but when answering using verbal and table representations, there were a number of mistakes in the answer. For a clearer picture, the representation and thought process of participant FA's problem solving are presented in Table 4.

Table 4. Analysis of results of participant with code FA.

Thought Process	Representation		
	Symbolic	Verbal	Table
Procedure-oriented	Able to manipulate symbols to complete the results comprehensively and systematically.	Unable to use the procedure appropriately.	Unable to use the proper procedure to complete the results.
Process-oriented	Able to interpret the meaning of written symbols.	Unable to implement the problem-solving process properly.	Unable to understand and implement the problem solving process.
Math object-oriented	Able to operate the relevant mathematical symbols.	Able to use words in understanding the mathematical objects.	Unable to interpret the problem in the form of mathematical tables.
Concept-oriented	Able to relate procedures and processes applied to symbolic representation in	Unable to relate procedures and processes applied to verbal representation in mathematical concepts	Unable to relate procedures and processes applied to precise table representation and unable to identify and operate

Thought Process	Representation		
	Symbolic	Verbal	Table
	mathematical concepts and able to identify concepts used in mathematical operations.	but able to identify concepts used in mathematical operations.	mathematical concepts.

The participant with code DP solved the two-variable linear equation problem using two representations, namely symbolic - verbal representations. When answering with the use of symbolic representation, it appeared that the participant was able to use the relevant symbols, namely variables x and y, and able to complete the process well even though the procedure used was not yet complete. The participant was able to interpret the symbols used, able to relate the procedures and processes applied to symbolic representation in the mathematical concepts, and able to identify the concepts used in the mathematical operations. Participant DP was able to answer correctly and was aware of the mistakes made in the completion process using verbal representation because of his understanding of the characteristics of the two-variable linear equation system. The representation and thought process of participant DP's problem-solving are presented in Table 5.

Table 5. Analysis of results of participant with code DP.

Thought Process	Representation	
	Symbolic	Verbal
Procedure-oriented	Able to manipulate symbols in the results although still incomplete.	Able to use appropriate and systematic procedures.
Process-oriented	Able to interpret the meaning of written symbols.	Able to apply the process correctly.
Math object-oriented	Able to operate the relevant mathematical symbols.	Able to use words in understanding the mathematical objects used.
Concept-oriented	Able to relate procedures and processes applied to symbolic representation in mathematical concepts and able to identify concepts used in mathematical operations.	Able to relate procedures and processes applied to verbal representation in mathematical concepts and able to identify concepts used in mathematical operations.

Table 6. Analysis of results of participant with code MN.

Thought Process	Symbolic Representation
Procedure-oriented	Unable to manipulate symbols to complete the results comprehensively and systematically.
Process-oriented	Able to interpret the meaning of written symbols.
Math object-oriented	Able to operate the relevant mathematical symbols.
Concept-oriented	Unable to relate processes and procedures applied to symbolic representation in mathematical concepts but able to identify concepts used in mathematical operations.

The participant with the code MN solved the problem of a two-variable linear equation system using symbolic representation. This is evident from the use of variables x and y. Participant MN was unable to use the proper procedures and processes because there was still a large amount of verbal representation. The participant was essentially able to interpret the symbols used but was unable to relate the procedures and processes applied to the symbolic representation in mathematical concepts though able to identify the concepts used in mathematical operations. The representation and the problem-solving thought process of the participant with code DP are presented in Table 6.

Discussion

The results of this research show that students essentially have the ability to solve problems using various types of representation. It was found that only 8.33% of students solved the problem presented using only a single type of symbolic representation. Students who had a high level of independence were seen to use a number of different types of representation, unlike other students who used only a single type of representation to solve the problem. In addition, the students' understanding of mathematical concepts was reflected in the greater number of representations used to solve the problem (Brenner et al., 1997). However, it should be noted that some of the students still encountered difficulties in the process of transferring the use of different types of representation, for example, in the beginning they used symbolic representation, then verbal or image representation (other types of representation). This is in line with the research of Gagatsis & Elia (2004) which concludes that students encounter numerous difficulties in changing from symbolic representation (algebra) into verbal representation. An investigation by Janvier (1987) shows how a successful transfer between one type of representation and another varies depending on the nature of the relationship between the chosen representations. Thus, in order to use different representations, students require knowledge of the subject matter as well as knowledge about representation because not all material can be presented using all types of representation.

This provides an overview of the importance of the ability to understand concepts and relationships between mathematical concepts, which are the main requirements for achieving multi-mathematical representation capability. The fact that some students were incapable of creating a proper representation for understanding the problem is believed to be because teachers, especially in Indonesia, often use only symbolic representation for solving mathematical problems, including two-variable linear equation problems. It should be recognized that the use of different representations makes concepts more accessible and enjoyable for students, and for this reason, solving the same problem using different representations or multiple representations shows better results than the use of a single representation (Hiebert & Carpenter 1992; Romberg & Kaput, 1999). Ainsworth (2006); Bransford & Schwartz (1999); and Keller & Hirsch (1998) state that two representations are better than one because they avoid the limitations of a single type of representation and build a new, clearer representation for the problem-solving process. This means that the ability to understand concepts and relationships between mathematical concepts is a necessary condition for the achievement of multi-mathematical representation capability, and multiple-representation ability itself is able to support students in understanding each different mathematical concept and anticipating mistakes in understanding mathematical concepts.

Conclusions

Students tend towards the use of symbolic representation rather than other types of representation in solving two-variable linear equation problems. When students use verbal representation, they are inclined to have difficulty in composing words while in image or table representation they are unable to solve the problem correctly. Based on these facts, we can conclude that the choice of representation used to solve a problem greatly affects the students' answers because the use of a particular type of representation requires knowledge of the mathematical concept as well as an understanding of the representation itself. Suggestions for further research include an exploration of the students' thought process in transitioning between different types of representation, such as verbal representation to symbolic representation or vice versa.

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